MATH 402 Worksheet 5

Friday 9/23/16

Recall that if we choose a coordinate system, we can represent any Euclidean isometry f as a 3×3-matrix A_f, such that:

- If f fixes the origin, then $A_f = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}$.
- If f is a translation by vector (v_1, v_2), then $A_f = \begin{pmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{pmatrix}$.
- If f is the reflection about the x-axis, then $A_f = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.
- If f is rotation of ϕ degrees about the origin, then $A_f = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(1) How can you represent the following isometries? Give a procedure, not necessarily the explicit matrix.
  (a) Rotation by ϕ about a point $A \neq O$.
  (b) Reflection about the line through the origin which makes an angle ϕ with the x-axis.
  (c) Reflection about a general line.

(2) What kind of isometry can be the composition $f = T_{\vec{v}} \circ R_{A,\alpha}$ of a rotation followed by a translation?
  Hint: Choose a coordinate system in which A is the origin. Use matrices or other methods to find if f can have fixed points (and how many). Then use vectors to determine the details.

(3) Let $R_{A,\alpha}$ be the rotation about a point A by an angle $\alpha \neq 0$, and $R_{B,\beta}$ be the rotation about a point $B \neq A$ by an angle $\alpha \neq 0$. What kind of isometry can the composition $R_{B,\beta} \circ R_{A,\alpha}$ be?
  Hints:
  - *Choose a coordinate system in which A is the origin.*
  - *Write $R_{B,\beta}$ as a composition of translations and a rotation about the origin.*
  - *How can you write a composition $R_{X,\phi} \circ T_{\vec{v}}$ as $T_{\vec{v}'} \circ R_{X,\phi}$?*