MATH 402 Worksheet 3

Friday 9/9/16

A line l is said to be *tangent* to a circle $\mathcal{C}$ if the intersection $l \cap \mathcal{C}$ consists of a single point T. In this situation, we say l is tangent to $\mathcal{C}$ *at* T.
Two distinct circles $\mathcal{C}_1$ and $\mathcal{C}_2$ are *mutually tangent* at a point T if the same line through T is tangent to both $\mathcal{C}_1$ and $\mathcal{C}_2$.
Prove the given facts / answer the questions:

(1) Given a circle $\mathcal{C}$ with center O and radius $\overline{OT}$, a line l is tangent to $\mathcal{C}$ at T if and only if l is perpendicular to OT.
(2) Given a circle $\mathcal{C}$ with center O and radius $\overline{OA}$, and given a point P outside of $\mathcal{C}$, there are exactly two tangent lines to $\mathcal{C}$ which pass through P.
(3) Suppose $\mathcal{C}_1$ and $\mathcal{C}_2$, with centers O_1 and O_2, are mutually tangent at T. Then the line O_1O_2 passes through T.
(4) Given a line l and a circle $\mathcal{C}$, how many points can there be in the intersection $l \cap \mathcal{C}$? Can you prove your answer?
(5) Given two circles $\mathcal{C}_1$ and $\mathcal{C}_2$, how many points can there be in the intersection $\mathcal{C}_1 \cap \mathcal{C}_2$?
(6) What does it mean for a point to be inside the circle? Justify the meaningfulness of your answer: can you prove that if P is a point inside $\mathcal{C}$ and Q is a point outside of $\mathcal{C}$, then the segment $\overline{PQ}$ intersects $\mathcal{C}$?