MATH 402 Worksheet 1

Friday 8/26/16

We are studying the system of a small experimental school, which would like to implement the following rules in the new semester:

(A) There will be 5 offered courses: art, biology, history, math, sociology.
(B) To maximize focus, every student will take exactly two courses.
(C) To maximize interaction among students, every pair of courses will have exactly one common student.

Call this "axiomatic system" S_5. Find a way to geometrically model this system, and then answer the following questions (and prove your answers using only the given axioms (A)-(C)).

(1) For the new scheme to work, how many students ought to be enrolled in the school?
(2) Show that any two students can have at most one course in common.
(3) How many students are in each class?
(4) How would the answers to the previous questions change if there are n offered courses, where n is an arbitrary natural number? Call such a system S_n.
(5) Make a sketch for S_3, S_4, S_5.
(6) Is S_n consistent? Complete? Are its axioms independent?

Bonus problem: *(If you don't have time in class, work on this problem at home.)*
Consider the axiomatic system S defined as follows:

Terms. The undefined terms are points, and a line is defined as a subset of points.
(S1) There are finitely many points.
(S2) Any two different points belong to an exactly one line.
(S3) Any two different lines have exactly one point in common.
(S4) There exist four points such that any three of them do not belong to the same line.

From S define a new system T, in such that

- the points of S are the lines of the new system T, and vice versa,
- the lines of S are the points of the new system T.

For this to make sense, we also suitably adjust the notion of "belonging:" If a point p belonged to a line l in S, the line corresponding to p in T now contains the point corresponding to l.

(1) Show that T also satisfies Axioms S1-S4 above.
(2) Show that Axioms S1-S4 are independent from each other. Note that this means you need to show the following four sub-statements:
    - Axiom 1 is independent from Axioms 2-4.
    - Axiom 2 is independent from Axioms 1,3,4.
    - Axiom 3 is independent from Axioms 1,2,4.
    - Axiom 4 is independent from Axioms 1-3.