MATH 402 Practice for second exam

Topics: material covered in class (through lectures, worksheets, projects, or homework) by the exam date. This corresponds to sections 1.7, 6.1, 6.2, 6.3, 6.4, 7.1, 7.2, 7.3, 7.4, 7.5.

1. All problems from homework, worksheet, and projects, as well as other problems from the book.

2. Definitions and statements

   (a) Define the order of a symmetry.

   (b) Define frieze and wallpaper groups.

   (c) Define points and lines in the Poincaré disk model of hyperbolic geometry.

   (d) Define points and lines in the Klein disk model of hyperbolic geometry.

   (e) Define right angles in the Klein disk, and sketch some perpendicular lines.

   (f) Define limiting parallels in the hyperbolic plane.

   (g) State the axiom of parallel lines in hyperbolic geometry.

   (h) State the fundamental theorem of parallels in hyperbolic geometry.

   (i) Define hyperbolic omega points in a model independent way.

   (j) State the basic properties of omega triangles (Pasch's axiom, exterior angle theorem, congruence theorem.)

   (k) Define Saccheri and Lambert quadrilaterals. Sketch a Saccheri and a Lambert quadrilateral in the Poincaré as well as Klein model.

   (l) Define the defect of a hyperbolic triangle.

3. Proofs and other longer answer questions

   (a) Show that all rotations in a finite symmetry group must have the same center of rotation.

   (b) Find the symmetry group of an equilateral triangle, and that of an isosceles triangle which is not equilateral.

   (c) Be able to describe the elements of the symmetry group of a given frieze or wallpaper pattern.

   (d) Suppose G is a frieze group with fundamental translation $T_{\vec{v}}$, and suppose l is a line invariant under $T_{\vec{v}}$. Prove that for any isometry $g \in G$, the line $g(l)$ is parallel to l or equal to l.

   (e) Show that the summit angles of a Saccheri quadrilateral are equal and acute.

   (f) Do there exist hyperbolic lines l and m with the property that the distance from a point of m to l is the same for any choice of a point on m?

   (g) Show that the angle sum of the two interior angles of an omega triangle is less than $180°$.

   (h) Show that limiting parallels cannot have a common perpendicular.

   (i) Prove that the Poincaré disk satisfies Hilbert's incidence axioms. Same for the Klein disk.

   (j) Exercises 7.3.3, 7.3.4, 7.3.5 (also all other exercises from 7.1-7.5).

4. True or false

   (a) The symmetries of a line form a frieze group.

   (b) Rectangles can sometimes be constructed in the Poincaré disk.

   (c) Rectangles can sometimes be constructed in the Klein disk.

   (d) Let $\triangle ABC$ and $\triangle XYZ$ be non-similar hyperbolic triangles. The sum of the interior angles of $\triangle ABC$ equals the sum of the interior angles of $\triangle XYZ$.

   (e) Hyperbolic right triangles have zero defect.

   (f) Hyperbolic circles do not always have tangent hyperbolic lines.

   (g) Reflections in the Klein disk do not preserve hyperbolic angles.

   (h) There are two non-congruent equilateral triangles in the Poincaré disk.