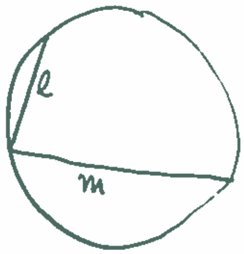
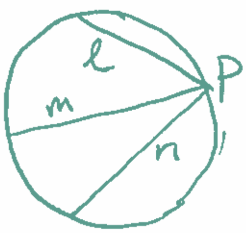
(1)

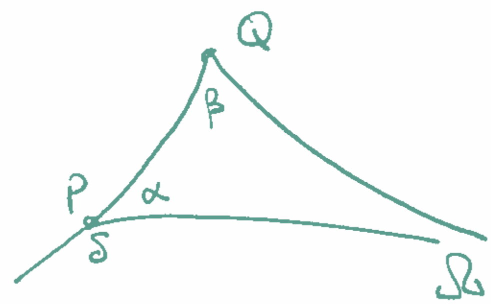


In the Klein model, lines are Euclidean chords; l is a limiting parallel to m iff l and m have Euclidean extensions which intersect on the boundary of the defining circle. Since intersection of (Euclidean) lines is symmetric (i.e. l intersects m $\Leftrightarrow$ m intersects l), we get that for lines in the Klein model, l is limiting to m $\Leftrightarrow$ m is limiting to l.



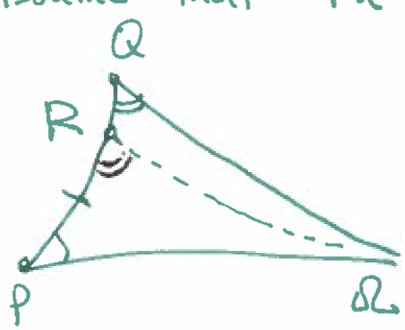
If l & m are limiting parallels to n in the same direction, this means that the Euclidean extensions of l & n intersect at P (a point on the defining circle), and the Euclidean extensions of m & n intersect at the same point P. Hence l & m intersect at P.

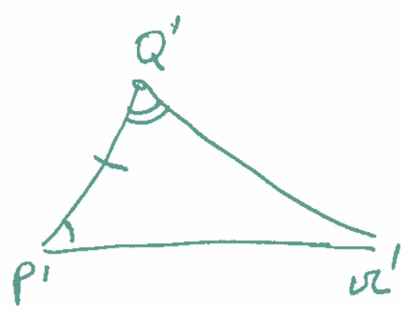
(2) Let α be the interior angle at P, β the interior angle at Q, and δ the exterior angle at P.



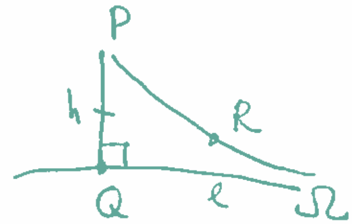
By the exterior angle theorem for Ω-Δ's, $\delta > \beta$. Hence,
$\alpha + \beta < \alpha + \delta = 180°$, since α & δ are supplementary.

(3) Assume that $\overline{PQ} > \overline{P'Q'}$, and let R be the point

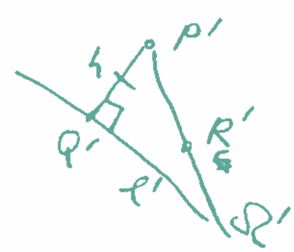




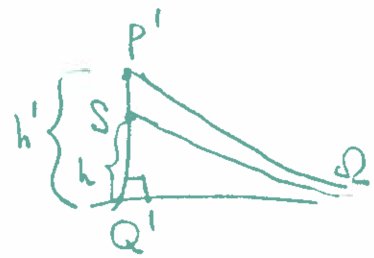
on $\overline{PQ}$ s.t. $\overline{PR} \cong \overline{P'Q'}$. Consider the omega triangles $\triangle PR\Omega$ and $\triangle P'Q'\Omega'$. By the congruence theorem 7.8, we get that $\angle PR\Omega \cong \angle P'Q'\Omega'$. But then the angle sum of $\triangle RQ\Omega$ is
$\angle PQ\Omega + \angle QR\Omega = \angle P'Q'\Omega' + 180° - \angle P'Q'\Omega' = 180°$,
contradicting the previous exercise.



4 Suppose $\overline{P'Q'}$ is another segment of length h, $\ell' \perp \overline{P'Q'}$ at Q', and $P'\Omega'$ is a limiting parallel to ℓ' at P'. Then the omega triangles $\triangle PQ\Omega$ & $\triangle P'Q'\Omega'$ satisfy the conditions of the congruence theorem 7.8, so $a(h)$ equals $\angle QP\Omega \cong \angle Q'P'\Omega'$, i.e, $a(h)$ is well-defined.



Now, suppose $h < h'$. Let S be the point on $\overline{P'Q'}$ s.t. $\overline{SQ'} = h$. Then $a(h) = \angle Q'S\Omega'$ and $a(h') = \angle Q'P'\Omega'$. By exercise 3 for $\triangle P'S\Omega'$, we have $\angle SP'\Omega' + \angle P'S\Omega' < 180°$, hence $a(h') + (180° - a(h)) < 180°$, i.e. $a(h') < a(h)$.



Finally, we need to show that if $a(h) = a(h')$, then $h = h'$. If $h < h'$, then $a(h') < a(h)$ by the above, so $h' \geq h$. Conversely, if $h' > h$, then $a(h) < a(h')$, so $h \geq h'$. Hence h & h' must be equal.