MATH 402 Non-Euclidean Geometry

Worksheet 3 on Euclidean transformations

Wednesday 09/30.

Let f be a transformation of the plane. We say that a point P is a *fixed point* of f is $f(P) = P$. Let's explore the properties of f as they relate to its fixed points.

Assume f is an isometry throughout.

1. Show that if A, B are two different fixed points of f, then every point on the line AB is also a fixed point of f.

   An isometry of this sort, i.e. one which has two different fixed points A, B, is called a *reflection.* The line AB is called the *axis* of reflection of f.

2. Suppose f is a reflection about an axis AB, and P is a point not on the line AB. Assume P is not fixed by f. What can you say about the relationship between the line AB and the line $Pf(P)$? First guess some claims, then prove them.

   Be careful: do not assume, unless you prove it first, that f "flips" the halves of the plane determined by AB.

3. Given an axis AB, how many different reflections about AB are there? Given a point P, how can you construct the image of P under a reflection?

4. Next suppose we are given two different points P, Q, but no f, and ask whether we can always find/define a reflection f for which $Q = f(P)$? What do you think? Guess an answer and then prove it.

5. Now go back again to f being any isometry, not necessarily a reflection. Show that if A, B, C are three non-collinear points, all fixed by f, then f is the identity.

6. Show that if f and g are two isometries that agree on three non-collinear points, then they agree on all points.