MATH 402 Non-Euclidean Geometry

Worksheet 2 on hyperbolic geometry

Monday 09/21.

In the last project, you learned how to construct a hyperbolic line through two given points in the Poincaré disk (which are not on the same diameter). Now, prove that there exists *at most one* such line as follows.

- In this part, we're in Euclidean geometry. Let $\mathcal{C}$ and $\mathcal{D}$ be circles which are orthogonal to each other. Let P be a point *inside* of $\mathcal{C}$ and *on* $\mathcal{D}$. Prove that the inverse P' of P with respect to $\mathcal{C}$ also lies *on* $\mathcal{D}$.

  Hints:

  - Form a point Q on the circle $\mathcal{D}$ which is a good candidate to be inverse to P. (*How?*)
  - Determine precisely what you need to show.
  - Label the centers and radii of the circles. Say O is the center and r the radius of $\mathcal{C}$ and O' is the center and r' the radius of $\mathcal{D}$.
  - Draw the line OO'.
  - Use our results on *power* of a point with respect to a circle.
  - Find a right triangle and use the Pythagorean theorem.

- Now, we go back to hyperbolic geometry. Let P, Q be two points in the Poincaré disk, not on the same diameter. Suppose there are two hyperbolic lines $\mathcal{D}$ and $\mathcal{D}'$ which contain P, Q. Show that $\mathcal{D} = \mathcal{D}'$.
  Hints:

  - Translate the problem into Euclidean geometry.
  - Use the previous result.
  - Use the theorem that three points determine a unique circle in Euclidean geometry.