MATH 402 Non-Euclidean Geometry

Exam 1 Practice Questions

Other great source for practice questions: the book. More precisely, any exercise or proof from Chapter 1, Chapter 2, and Section 7.2. In addition, review all homework; solutions are posted on the webpage.

On the actual exam, you will be given a copy of the statements of Euclid's and Hilbert's axioms.

1. Definitions and statements
   (a) State Playfair's postulate.
   (b) Define an angle.
   (c) Define congruent triangles.
   (d) Define similar triangles.
   (e) State Pasch's theorem/postulate.
   (f) State the vertical angle theorem.
   (g) State the exterior angle theorem.
   (h) State SAS/SSS/ASA congruence rule.
   (i) Define neutral or absolute geometry.
   (j) Define points and lines in the Poincaré disk model of hyperbolic geometry.
   (k) Define orthogonal circles.
   (l) Define the inverse of a point with respect to a given circle.
2. Euclidean constructions
   (a) Equilateral triangle with a given side.
   (b) Angle bisector.
   (c) Inverse of a point with respect to a circle.
   (d) Given a circle $\mathcal{C}$ and two points A, B, another circle $\mathcal{D}$ which is orthogonal to $\mathcal{C}$ and passes through the given points A, B.

3. Proofs

   (a) Prove the vertical angle theorem.

   (b) Prove the exterior angle theorem.

   (c) Let $\mathcal{C}$ be a circle with origin O. Prove that there isn't a circle $\mathcal{D}$ which goes through O and such that $\mathcal{C}$ and $\mathcal{D}$ are orthogonal.

   (d) Prove that Playfair's postulate implies the following statement: If l_1 and l_2 are two unequal parallel lines, and m is another line which intersects l_1 (but is not equal to l_1), then m also intersects l_2.

   (e) Given Hilbert's axioms, prove SSS.

   (f) Given Hilbert's axioms, prove ASA.

   (g) Consider the axiomatic system defined by the following. The undefined terms are points, and a line is defined as a set of points. The axioms are:

      i. There are exactly four points.

      ii. There are exactly four lines.

      iii. Given any two different points there exists at least one line that contains them.

      - I claim that this system is consistent. Give a model.
      - Show that each of these three axioms is independent from the others.
      - Is the system complete? Why or why not?
      - Is the following statement true or false for the system: Every line contains at most two points. Justify your answer.
      - From the given system S form a new one T which is "dual," i.e. points in T are lines in S, and lines in T are points in S. Does T satisfy the same axioms as S? If the answer is yes, prove your claim by proving each axiom holds for T. If the answer is no, prove that one of the axioms for S (which one?) does not hold.

(h) Let $\mathcal{C}$ be a circle, P a point inside of $\mathcal{C}$, and let $\mathcal{D}$ be a circle orthogonal to $\mathcal{C}$ which passes through P. Let P' be the inverse of P with respect to the circle $\mathcal{C}$. Show that P' must also lie on $\mathcal{D}$.

4. True or false?

   (a) Euclid's 1st/2nd/3rd/4th/5th postulate in Euclidean/spherical/hyperbolic geometry.

   (b) Euclid's 5th postulate is inconsistent with the other four.

   (c) Euclid's 5th postulate is independent from the other four.

   (d) Rectangles can always be constructed in the Poincaré disk.

   (e) Rectangles can sometimes be constructed in the Poincaré disk.

   (f) In Euclidean geometry, a line and a circle can have exactly one point of intersection.]

   (g) In Euclidean geometry, a given two parallel lines l_1 and l_2, there exists a unique line m which is perpendicular to both l_1 and l_2.

   (h) The exterior angle theorem is true in Euclidean geometry.

   (i) The exterior angle theorem is true in hyperbolic geometry.

   (j) The sum of the interior angles of a hyperbolic triangle is $180°$.