MATH 402 Non-Euclidean Geometry

Homework - Week 4

Due Friday 09/18.

1. (a) Show that if A, B, C are three distinct points on a given circle, then they cannot be collinear.

   (b) Using (a), show that a line l and a circle $\mathcal{S}$ can have *at most two* intersection points.

Recall the definition of tangency: l is tangent to $\mathcal{S}$ if the intersection $l \cap \mathcal{S}$ has exactly one point.

2. Let $\mathcal{S}$ be a circle, and let A be a point on $\mathcal{S}$. Show that there exists *at most one* tangent line of $\mathcal{S}$ through l.

   Remark: We showed that there exists at least one tangent in class, so after this exercise we'll be able to conclude that there exists a unique tangent line.

3. As we did in class, we can define the angle between a circle $\mathcal{S}$ and a line l which intersects $\mathcal{S}$ at two points A, B one of two ways:

   (a) Take the line l_A through A tangent to $\mathcal{S}$ (unique by (2)). Take the angle of intersection between l and l_A (i.e. the angle formed by l and l_A which is less than or equal to a right angle.) Call this angle α_A.

   (b) Take the line l_B through B tangent to $\mathcal{S}$. Take the angle of intersection between l and l_B (i.e. the angle formed by l and l_B which is less than or equal to a right angle.) Call this angle α_B.

   Show that these two definitions give the same angle, i.e. show that $\alpha_A = \alpha_B$.

4. This problem deals with *similar* triangles.

   (a) Define what it means for two triangles ΔABC and ΔXYZ to be similar.

   (b) Read and understand the proof of Theorem 2.25 from the book. Then close the book and write it down in your own words. *You* don't *have to submit the proof, and if you do, it won't be graded.*

   (c) Do Exercise 2.5.4 from the book.