Homework for Friday 09/04.

1. Rewrite your solution to last week’s problem posted online, while fixing your mistakes. Recall, the question was: **Is the following axiomatic system consistent?** If yes, give a model; if not, show why. *This time you know (from class) that the system is consistent, and there exists a model with more than four but less than ten points.*

   **Terms.** The undefined terms are points, and a line is defined as a subset of points.

   **Axiom 1.** There are finitely many points.

   **Axiom 2.** Any two different points belong to an exactly one line.

   **Axiom 3.** Any two different lines have exactly one point in common.

   **Axiom 4.** There exist four points such that any three of them do not belong to the same line.

2. Let $S$ be an axiomatic system satisfying the Axioms 1-4 above. From $S$ define a new system $T$, in such that
   - the points of $S$ are the lines of the new system $T$, and vice versa,
   - the lines of $S$ are the points of the new system $T$.

   For this to make sense, we also suitably adjust the notion of “belonging.” If a point $p$ belonged to a line $l$ in $S$, the line corresponding to $p$ in $T$ now contains the point corresponding to $l$.

   **Show that $T$ also satisfies Axioms 1-4 above.**

3. **Show that Axioms 1-4 are independent from each other.** Note that this means you need to show the following four sub-statements:
   - Axiom 1 is independent from Axioms 2-4.
   - Axiom 2 is independent from Axioms 1,3,4.
   - Axiom 3 is independent from Axioms 1,2,4.
   - Axiom 4 is independent from Axioms 1-3.

4. **Read carefully Hilbert’s Axioms** (given in Appendix D). For each one of them, do the following:
   - **Make a little sketch** to illustrate it,
   - **Give the equivalent statement in Euclid’s system** (it may be an axiom or a theorem).