MATH 402 Non-Euclidean Geometry
Homework - Week 2

Homework for Friday 09/04.

1. Rewrite your solution to last week's problem posted online, while fixing your mistakes. Recall, the question was: **Is the following axiomatic system consistent?** If yes, give a model; if not, show why. *This time you know (from class) that the system is consistent, and there exists a model with more than four but less than ten points.*

Terms. The undefined terms are points, and a line is defined as a subset of points.

Axiom 1. There are finitely many points.

Axiom 2. Any two different points belong to an exactly one line.

Axiom 3. Any two different lines have exactly one point in common.

Axiom 4. There exist four points such that any three of them do not belong to the same line.

2. Let S be an axiomatic system satisfying the Axioms 1-4 above. From S define a new system T, in such that

   - the points of S are the lines of the new system T, and vice versa,
   - the lines of S are the points of the new system T.

   For this to make sense, we also suitably adjust the notion of "belonging:" If a point p belonged to a line l in S, the line corresponding to p in T now contains the point corresponding to l.

   Show that T also satisfies Axioms 1-4 above.

3. **Show that Axioms 1-4 are independent from each other.** Note that this means you need to show the following four sub-statements:

   - Axiom 1 is independent from Axioms 2-4.
   - Axiom 2 is independent from Axioms 1,3,4.
   - Axiom 3 is independent from Axioms 1,2,4.
   - Axiom 4 is independent from Axioms 1-3.

4. **Read carefully Hilbert's Axioms** (given in Appendix D). For each one of them, do the following:

   - **Make a little sketch** to illustrate it,
   - **Give the equivalent statement in Euclid's system** (it may be an axiom or a theorem).