1 (Holmes, 1.2). For a pendulum that starts from the rest, the period $p$ depends on the length $\ell$ of the rod, gravity $g$, the mass $m$ of the ball, and the initial angle $\theta_0$ at which the pendulum starts.

(a) Use dimensional analysis to determine the functional dependence of $p$ on these four quantities.

(b) For the largest pendulum ever built, the rod is 70 ft and the ball weighs 900 lbs. Assume $\theta_0 = \pi/6$ and explain how to use a pendulum that fits on your desk to determine the the period of this largest pendulum.

(c) Suppose it is found that $p$ depends linearly on $\theta_0$ with $p = 0$ for $\theta_0 = 0$. What does your result in part (a) reduces to in this case?

2. “Prove” the Pythagoras theorem using dimensional analysis.

(Remark. See Barenblatt, Scaling, self-similarity, and intermediate asymptotics, Section 1.2.3. In Migidal’s book, for instance, a proof was presented after the theorem in rigorous geometry courses. In any case, Barenblatt* did not recommend that this proof replace those used in geometry class, and I, either. But, it is an entertaining application of dimensional analysis.)

3 (Holmes, 1.26). One of the standard experimental tests used in the study of fluid motion through porous materials consists of determining the displacement $u$ when the material is given a constant load. The governing equation in this case is

$$u_t = H(1 + u_x^3)u_{xx}.$$  

The initial condition is $u(x,0) = 0$, the boundary conditions are $u_x(0,t) = -1$, and $u(\infty, t) = 0$.

(a) What are the dimensions of the constant $H$?

(b) Find a dimensionally reduced form for the solution and then use this to transform the above diffusion problem into one involving a nonlinear ordinary differential equation. Make sure to state what happens to the initial and boundary conditions. You do not have to solve this problem.

(c) In the experiment, the surface displacement $u(0,t)$ is measured. Without solving the problem, use your result from $b$ to sketch $u(0,t)$ as a function of $t$.

(d) Suppose the experimental data show that $u(0,t) = 16t$ cm/sec. Using your result from part (c), explain why the mathematical model is incorrect. Also, explain why changing the differential equation to either $u_t = Hu_{xx}$ or to $u_t = H(1 + u_x^5)u_{xx}$ will also produce an incorrect model.

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*Requiesce in pace.