1. Prove that $\sin(kx) \to 0$ as $k \to \infty$ in $L^2(0,1)$. This problem illustrates that a weakly convergent sequence can be rather badly behaved.

2. Find $L = L(p, z, x)$ so that the PDE

$$-\Delta u + D\phi \cdot Du = f \quad \text{in } U$$

is the Euler-Lagrange equation corresponding to the functional $I[w] := \int_U L(Dw, w, x) \, dx$. (Hint: Look for a Lagrangian with an exponential term.)