\[ u(x, t) = u_x(x, t) = 0 \quad \text{Neumann condition} \]

(E): \[
-\dddot{x} = \lambda \ddot{x}, \quad 0 < x < l.
\]
\[
x(0) = x(l) = 0, \quad x'(0) = x'(l).
\]

Claim. \( \lambda > 0 \) (real).

\[
\left( x \right) \left( \int_0^l \dddot{x} \, dx \right) = \int_0^l \dddot{x} \, dx = \lambda \int_0^l \ddot{x} \, dx = \lambda \int_0^l \frac{dx}{t} \, dx.
\]

\[
\int_0^l \ddot{x} \, dx = \lambda \int_0^l \frac{dx}{t} \, dx \quad \Rightarrow \quad \lambda > 0.
\]

Case 1. \( \lambda > 0 \).

Let \( \lambda = \beta^2 > 0 \) \( \Rightarrow \)
\[
x(x) = C \cos \beta x + D \sin \beta x.
\]
\[
\ddot{x}(0) = D\beta = 0 \quad \Rightarrow D = 0
\]
\[
\ddot{x}(l) = -C\beta \sin \beta l = 0 \quad \Rightarrow \beta l = n\pi.
\]

\[
\text{E-Value: } \lambda(n) = \left( \frac{n\pi}{l} \right)^2, \quad n = 1, 2, \ldots
\]

E-Func: \( \cos \frac{n\pi x}{l} \)

Case 2. \( \lambda = 0 \).

\( \Rightarrow \) \( x(x) = C + DX \).
\[
\ddot{x}(D) = 0 \quad \Rightarrow \quad x(0) = \text{const.}
\]

\[
\sum_{n=0}^{\infty} \lambda_n = \left( \frac{n\pi}{l} \right)^2, \quad n = 0, 1, 2, \ldots
\]
\[
X_n = \cos \frac{n\pi x}{l}.
\]
DIFFUSION EQUATION \[ T(t) = A.e^{-\lambda t} \quad T_n(t) = A_n e^{-\left(\frac{nt}{l}\right)^2} t. \quad n \geq 1. \]

\[ \phi(x) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} A_n e^{-\frac{(nt)^2}{l}} \cos \frac{nx}{l}. \]

\[ u(x,t) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{nx}{l} e^{-\frac{(nt)^2}{l}}. \]

UNWE EQUATION \[ T_n(t) = A_n \cos \frac{\frac{nt}{l} \cdot \phi}{q} + B_n \sin \frac{\frac{nt}{l} \cdot \phi}{q}. \quad n \geq 1. \]

\[ \phi(x) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{nx}{l}. \]

\[ u(x,t) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} B_n \sin \frac{\frac{nt}{l} \cdot \phi}{q} \cos \frac{nx}{l}. \]

\[ \phi(x) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} B_n \sin \frac{nx}{l}. \]

\[ u(x,t) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} B_n \sin \frac{\frac{nt}{l} \cdot \phi}{q} \cos \frac{nx}{l}. \]

\[ \psi(x) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} B_n \sin \frac{nx}{l}. \]

\[ U(0,t) = 0 = U_x(l,t) \quad \text{(MIXED)} \]

\[ \begin{cases} -x'' = \lambda x & 0 < x < l \quad \text{(HIV)} \\ x(0) = 0 = x'(l) \end{cases} \]

SCHRÖDINGER EQUATION \[ u_t = uu_{xx} \quad 0 < x < l \]

SEPARATION OF VARIABLES \[ u(x,t) = X(x)T(t) \]

\[ \Rightarrow \quad \frac{T'}{T} = \frac{X''}{X} = \lambda \quad \Rightarrow \quad T(t) = A e^{-\lambda t} \quad (HIV) \]