**Well-posed**:

- **Existence**
  - (Obvious)
  - (Too few conditions ⇒ non-unique)

- **Uniqueness**
  - (Too many ⇒ non-exist.)

- **Stability**: Solution depends on data continuously. (Physically relevant)

**Examples** 1. Vibrating string with external force.

\[ \begin{align*}
    u_{tt} - c^2 u_{xx} &= f(x,t) & \text{for } 0 < x < c &\text{ & } t > 0, \\
    u(x,0) &= \phi(x), & u_t(x,0) &= \psi(x), \\
    u(0,t) &= g(t), & u(c,t) &= h(t) .
\end{align*} \]

**Existence**: For arbitrary \( f, \phi, \psi, g, h \), \( \exists \, u = \text{solution} \).

**Uniqueness**: \( \equiv \).

**Stability**: \( f_1, \phi_1, \psi_1, g_1, h_1 \), \( \forall \, f_2, \phi_2, \psi_2, g_2, h_2 \Rightarrow u_1 \approx u_2 \).

Requires definition of "closeness"; topology.

Well-posed for appropriate closeness. (Later)

2. Diffusion

\[ \begin{align*}
    u_{tt} - c^2 u_{xx} &= 0 & -\infty < x < \infty, \\
    u(x,0) &= f(x).
\end{align*} \]

Well-posed for \( t > 0 \), but not for \( t < 0 \)!

3. \( u_{xx} + u_{yy} = 0 \) for \( -\infty < x < \infty, 0 < y < \infty \)

\[ \begin{align*}
    u(x,0) &= 0, \\
    u_y(x,0) &= e^{\sqrt{2}n} \sin \pi x & n \in \mathbb{Z}.
\end{align*} \]

\( \Rightarrow \, u_{n}(x) = \frac{1}{\sqrt{\pi}} e^{\sqrt{2}n} \sin \pi x \sin n y \) (Check!)

Data. \( u(x,0), \frac{\partial u}{\partial y} (x,0) \to 0 \) as \( n \to \infty \).

Solution \( u_n(x,y) \to 0 \) as \( n \to \infty \) for \( y > 0 \), not well-posed!