1. (a) Verify that \( y = x^a \) solves the differential equation \( x^2 y'' = 2y \) if the constant \( a \) satisfies the equation \( a^2 - a - 2 = 0 \). Thus get the two solutions \( x^2 \) and \( x^{-1} \). Note that the first is valid on the whole interval \(-\infty < x < \infty\) but the second on \(-\infty < x < 0\) or \(0 < x < \infty\) only.

(This behavior is typical for a broad class of linear homogeneous equations known as equations of Euler type. For this class the substitution \( y = x^a \) always lead to an algebraic equation for \( a \).)

(b) The equation \( x^2 y''' = 2y'' \) admits a solution \( y = x^a \), where \( a \) is a nonzero constant. What are the possible values of \( a \)?

2. Boyce and DiPrima, Section 2.2, #2, #5, #6, #8.

3. Boyce and DiPrima, Section 2.2, #9(a)(c), #14(a)(c).

4. For each of the following ODE’s, draw a direction field for the given differential equation by using about five isoclines. Then, sketch in some integral curves, using the information provided by the direction field. Do whatever else is asked.

(a) \( y' = -y/x \); solve the equation exactly and compare your integral curves with the correct ones.

(b) \( y' = x - y \); find a solution whose graph is also an isocline, and verify this fact analytically.

(c) \( y' = x^2 + y^2 - 1 \)

(d) \( y' = \frac{1}{x + y} \); use the interval \(-3\) to \(3\) on both axes; draw in the integral curves that pass respectively through \((0,0), (-1,1), (0,-2)\). Will these curves cross the line \( y = -x - 1 \)? Explain by using properties of integral curves.

5. Sketch a direction field, concentrating on the first quadrant for \( y' = \frac{-y}{x^2 + y^2} \). Explain, using it and the ODE itself how one can tell that the solution \( y(x) \) satisfying the initial condition \( y(0) = 1 \)

(a) is a decreasing function for \( x > 0 \);

(b) is always positive for \( x > 0 \).