HW 5, due Monday, February 25: problems 4(a) and 9 on p. 45 of the book. (For Friday, February 22, do the problems 1, 3, 7,10 on pp. 44, 45, but don't hand them in.)

Solution of 4(a). Consider the subsets of Γ that prove the same propositions as Γ (equivalently, that prove all $\phi \in \Gamma$). Since Γ is finite, we can take among those subsets a minimal one and call it Δ (so no proper subset of Δ proves all $\phi \in \Gamma$). It is easy to see that then Δ is independent.

Solution of 9. We prove the contrapositive: Suppose that for every $m \geq 1$ we have $\nvdash \phi_1 \vee \cdots \vee \phi_m$. Then by the Completeness Theorem there is for every $m \geq 1$ a valuation v with $v(\phi_1) = \cdots = v(\phi_m) = 0$, that is, $v(\neg\phi_1) = \cdots = v(\neg\phi_m) = 1$, so by the result of problem 8 there is a v such that $v(\neg\phi_n) = 1$ for all $n \geq 1$, that is, $v(\phi_n) = 0$ for all $n \geq 1$.