HW 5, due Monday, February 25: problems 4(a) and 9 on p. 45 of the book. (For Friday, February 22, do the problems 1, 3, 7, 10 on pp. 44, 45, but don’t hand them in.)

Solution of 4(a). Consider the subsets of Γ that prove the same propositions as Γ (equivalently, that prove all φ ∈ Γ). Since Γ is finite, we can take among those subsets a minimal one and call it ∆ (so no proper subset of ∆ proves all φ ∈ Γ). It is easy to see that then ∆ is independent.

Solution of 9. We prove the contrapositive: Suppose that for every m ≥ 1 we have ̸⊢ φ₁ ∨ ... ∨ φₘ. Then by the Completeness Theorem there is for every m ≥ 1 a valuation v with v(ϕ₁) = ... = v(ϕₘ) = 0, that is, v(¬ϕ₁) = ... = v(¬ϕₘ) = 1, so by the result of problem 8 there is a v such that v(¬φₙ) = 1 for all n ≥ 1, that is, v(ϕₙ) = 0 for all n ≥ 1.