HW 12, due Monday April 22: problems 10 and 18 on pp. 110, 111 of the book. Let me add some clarifications to these problems. No class on Wednesday April 17.

Problem 10. Here $L = \{ f \}$ with $f$ a unary function symbol. A permutation of a set $A$ is by definition a bijection $b : A \rightarrow A$. In (i) you are asked to produce an $L$-sentence $\sigma_{\text{sur}}$ such that for all $L$-structures $\mathcal{A} = \langle A; f^A \rangle$,

$$\mathcal{A} \models \sigma_{\text{sur}} \iff f^\mathcal{A} : A \rightarrow A \text{ is surjective.}$$

In (iv) you need to find an $L$-sentence $\sigma$ such that (a) and (b) below hold:

(a) for all $L$-structures $\mathcal{A} = \langle A; \ldots \rangle$, if $\mathcal{A} \models \sigma$, then $A$ is infinite;

(b) for every infinite set $A$ there is a model $\mathcal{A} = \langle A; \ldots \rangle$ of $\sigma$.

Problem 18. Here $k$ is a fixed (but arbitrary) natural number $\geq 1$. Let the language $L$ have just a binary relation symbol $R$. Then a graph is an $L$-structure $\mathcal{G} = \langle G; R \rangle$ where $R$ is a symmetric irreflexive binary relation on the (nonempty) set $G$ whose elements are thought of as the vertices of the graph. A subgraph of such $\mathcal{G}$ is a graph $\mathcal{G}_0 = \langle G_0; R \cap G_0^2 \rangle$ with $G_0 \subseteq G$. A $k$-coloring of such a graph $\mathcal{G} = \langle G; R \rangle$ is by definition a function $c : G \rightarrow \{1, \ldots, k\}$ such that $c(g) \neq c(h)$ for all pairs $(g, h) \in R$. In the statement of the problem the language $L$ is augmented by $k$ extra unary predicate symbols $C_1, \ldots, C_k$, to give the language $L_k = L \cup \{C_1, \ldots, C_k\}$ and it is observed that $\mathcal{G}$ has a $k$-coloring iff $\mathcal{G}$ can be expanded to an $L_k$-structure that satisfies the sentences listed there involving the new symbols $C_1, \ldots, C_k$. Please check for yourself that this is a correct observation, and then solve the problem. Hint: given $\mathcal{G}$, you might consider extending $L_k$ further by names for the elements of $G$. 

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