HW 12, due Monday April 22 : problems 10 and 18 on pp. 110, 111 of the book. Let me add some clarifications to these problems. No class on Wednesday April 17.

Problem 10. Here $L = \{f\}$ with f a unary function symbol. A *permutation* of a set A is by definition a bijection $b : A \to A$. In (i) you are asked to produce an L-sentence σ_{sur} such that for all L-structures $\mathcal{A} = \langle A; f^{\mathcal{A}} \rangle$,

$$\mathcal{A} \models \sigma_{sur} \iff f^{\mathcal{A}} : A \to A \text{ is surjective.}$$

In (iv) you need to find an L-sentence σ such that (a) and (b) below hold:

(a) for all L-structures $\mathcal{A} = \langle A; \ldots \rangle$, if $\mathcal{A} \models \sigma$, then A is infinite;

(b) for every infinite set A there is a model $\mathcal{A} = \langle A; \ldots \rangle$ of σ.

Problem 18. Here k is a fixed (but arbitrary) natural number ≥ 1. Let the language L have just a binary relation symbol R. Then a graph is an L-structure $\mathcal{G} = \langle G; R \rangle$ where R is a symmetric irreflexive binary relation on the (nonempty) set G whose elements are thought of as the vertices of the graph. A *subgraph* of such $\mathcal{G}$ is a graph $\mathcal{G}_0 = \langle G_0; R \cap G_0^2 \}$ with $G_0 \subseteq G$. A k-coloring of such a graph $\mathcal{G} = \langle G; R \rangle$ is by definition a function $c : G \to \{1, \ldots, k\}$ such that $c(g) \neq c(h)$ for all pairs $(g, h) \in R$. In the statement of the problem the language L is augmented by k extra unary predicate symbols $C_1, \ldots, C_k$, to give the language $L_k = L \cup \{C_1, \ldots, C_k\}$ and it is observed that $\mathcal{G}$ has a k-coloring iff $\mathcal{G}$ can be expanded to an L_k-structure that satisfies the sentences listed there involving the new symbols $C_1, \ldots, C_k$. Please check for yourself that this is a correct observation, and then solve the problem. Hint: given $\mathcal{G}$, you might consider extending L_k further by names for the elements of G.