Conjunctive Normal Form and Disjunctive Normal Form. We give here a somewhat stronger form of the result that every proposition is equivalent to one in conjunctive normal form and to one in disjunctive normal form. We also give a different proof. First, for typographical reasons we use $\bigwedge$ and $\bigvee$ instead of the double bigwedge and double bigvee used in the book. Let $\phi = \phi(p_0, \ldots, p_n)$ be such that $v(\phi) = 1$ for at least one truth assignment $v : \{p_0, \ldots, p_n\} \to \{0, 1\}$. We claim that then ϕ is equivalent to a disjunctive normal form

$$(*) \qquad \bigvee_{i \le N} \bigwedge_{j \le n} p_j^{e_{ij}},$$

where all $e_{ij} \in \{-1, 1\}$ and $\psi^1 := \psi$ and $\psi^{-1} := \neg\psi$ for propositions ψ; moreover, $N \le 2^{n+1}$. To prove this, let $v_0, \ldots, v_N$ be the distinct truth assignments $v : \{p_0, \ldots, p_n\} \to \{0, 1\}$ such that $v(\phi) = 1$. For each $v = v_i$, set $e_{ij} := 1$ if $v(p_j) = 1$ and $e_{ij} = -1$ if $v(p_j) = 0$. It is then easy to check that ϕ is equivalent to the formula $(*)$ above: any truth assignment $v : \{p_0, \ldots, p_n\} \to \{0, 1\}$ gives the same value to ϕ as to the above disjunction.

There is one other case to consider, namely that $v(\phi) = 0$ for all truth assignments $v : \{p_0, \ldots, p_n\} \to \{0, 1\}$. But then ϕ is equivalent to $\perp$, which, being an atom, is a disjunctive normal form.

As to conjunctive normal forms: $\neg\phi$ is equivalent to a disjunctive normal form, so $\neg\neg\phi$ (and thus ϕ) is equivalent to a conjunctive normal form, using "De Morgan".