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For the convenience of the reader we reproduce the statements of the main theorem (Theorem 4.1) and one of the accompanying remarks (Remark 4.3) in the cited paper [2]. To clarify the remainder of this erratum we have divided the statement of the theorem into two parts.

We recall that $S_{\mathbf{a}}$, for a sequence $\mathbf{a} = (a_1, a_2, \ldots)$ with $a_j \in \{\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\}$, $j = 1, 2, 3, \ldots$, denotes the Sierpiński carpet obtained by starting from the fixed square $[0,1]^2$ (called the 0th level square), subdividing all $(j-1)$st level squares Q into congruent and essentially disjoint subsquares of side length a_j times the side length of Q, removing the central subsquare from each such subdivision, and passing to the limit as $j \to \infty$. For a more precise description of the procedure defining these carpets, see section 2 of the cited paper [2]. Theorem 4.1 of [2] refers only to the **self-similar** carpets $S_{\mathbf{a}}$ obtained for constant sequences $\mathbf{a}$. For instance $S_{\mathbf{3}}$ denotes the classical $\frac{1}{3}$ Sierpiński carpet. The set $\mathrm{Slopes}(S_{\mathbf{a}})$ denotes the set of slopes, in the interval $[0,1]$ of nontrivial line segments contained in $S_{\mathbf{a}}$.

Theorem 1 ([2], Theorem 4.1)**.** *(1) Let* $\mathbf{a} = (\frac{1}{a}, \frac{1}{a}, \frac{1}{a}, \ldots)$ *be a constant sequence. Then the set of slopes* $\mathrm{Slopes}(S_{\mathbf{a}})$ *is the union of the following two sets:*

$$A = \left\{ \frac{p}{q} \,:\, p+q \le a, \quad 0 \le p < q \le a-1, \quad p,q \in \mathbb{N} \cup \{0\}, \quad p+q \text{ odd} \right\}$$

and

$$B = \left\{ \frac{p}{q} \,:\, p+q \le a-1, \quad 1 \le p \le q \le a-2, \quad p,q \in \mathbb{N}, \quad p,q \text{ odd} \right\}.$$

(2) If $\alpha \in A$, then each nontrivial line segment in $S_{\mathbf{a}}$ with slope α touches vertices of peripheral squares, while if $\alpha \in B$, then each nontrivial line segment in $S_{\mathbf{a}}$ with slope α is disjoint from all peripheral squares. For each $\alpha \in A \cup B$, there exist maximal line segments in $S_{\mathbf{a}}$ with slope α. Finally, if $b < a$, then any maximal nontrivial line segment in $S_{\mathbf{b}}$ is also contained in $S_{\mathbf{a}}$. In particular, $\mathrm{Slopes}(S_{\mathbf{b}}) \subset \mathrm{Slopes}(S_{\mathbf{a}})$.

Remark 2 ([2], Remark 4.3)**.** *Fix* $\mathbf{a}$*, write* $\mathrm{Slopes}(S_{\mathbf{a}}) = A \cup B$ *as in the statement of Theorem 1, and fix $\alpha \in A \cup B$. If $\alpha \in A$, then there exists a line segment of slope α passing through the origin* $(0,0)$*. On the other hand, if $\alpha \in B$, then there exists a line segment of slope α passing through the midpoint* $(\frac{1}{2}, 0)$*. Other line segments of this slope are obtained by applying Euclidean translations.*

Part (1) of Theorem 1 provides a precise description of all possible slopes of nontrivial line segments in $S_{\mathbf{a}}$. The proof of this fact is divided into two parts. In the first part, we construct explicitly one segment with slope α for any $\alpha \in A \cup B$. As mentioned in Remark 2, if $\alpha \in A$, then there exists a line segment of slope α passing through the origin $(0,0)$ and if $\alpha \in B$, then there exists a line segment of slope α passing through the midpoint $(\frac{1}{2}, 0)$.

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In part (2) of Theorem 1 the statement *"If $\alpha \in A$, then each nontrivial line segment in $S_{\mathbf{a}}$ with slope α touches vertices of peripheral squares"* is incorrect. In [1] Chen and Niemeyer provide the following counterexample to the assertion: there exists a nontrivial line segment beginning at the origin $(0,0)$ with slope $\alpha = \frac{2}{3}$ in the carpet $S_{\mathbf{7}}$ which avoids all peripheral squares. Moreover, in Theorem 3.5 of [1] they provide a further refinement of part (2) of Theorem 1 which clarifies the different possibilities for segments with slope in the set A, emanating either from corners or from midpoints of constituent squares in the construction, and which also analyzes when such segments avoid or touch the vertices of peripheral squares. We remark that such a detailed analysis is of great importance to the authors of [1], who use it to initiate a theory of fractal billiards on the carpets $S_{\mathbf{a}}$.

In conclusion we would like to take the opportunity to clarify the final sentence of Remark 2. The assertion *"Other line segments of this slope are obtained by applying Euclidean translations"* means only that the carpet $S_{\mathbf{a}}$ contains additional line segments of slope α, which can (evidently) be obtained from segments of the indicated type by Euclidean translations. The precise location of these additional segments, of course, is dictated by the structure of the carpet itself.

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References

[1] J. Chen and R. Niemeyer, *Periodic billiard orbits of self-similar Sierpiński carpets*, preprint, 2013, arXiv:1303.4032.

[2] E. Durand Cartagena and J. T. Tyson, *Rectifiable curves in Sierpiński carpets*, Indiana Univ. Math. J., **60**, no. 1 (2011), 285–309.

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