GEOMETRIC ANALYSIS AND APPLICATIONS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
JULY 12–15, 2006

Schedule of the talks

Wednesday, 7/12, morning session, 314 Altgeld Hall.

8:45 Introductory remarks (the organizers), departmental welcome (J. Rosenblatt, Chair, Mathematics Department, UIUC)

9:00 – 10:00 M. Bonk, Michigan: *"Metrics on the boundary of negatively curved spaces"*

10:15 – 11:15 R. Brockett, Harvard: *"Some degenerate diffusion processes and random matrices associated with sub-Riemannian geometry"*

11:30 – 12:30 R. Serapioni, Trento: *"Intrinsic Lipschitz graphs in Heisenberg groups"*

12:30 – 2:30 Lunch

Wednesday, 7/12, afternoon session, 245 Altgeld Hall.

2:30 – 3:00 I. Hussein, Worcester Polytechnic Institute: *"Optimal control of second-order systems on Riemannian manifolds"*

3:10 – 3:40 C. Selby, Purdue: *"ϵ-approximation technique for determining geometric quantities for hypersurfaces in Carnot groups of step two"*

3:40 – 4:00 Coffee break

4:00 – 4:30 D. Vittone, Pisa: *"The Bernstein problem for intrinsic graphs in Heisenberg groups"*

4:40 – 5:10 V. Magnani, Pisa: *"Intrinsic blow-up of submanifolds in stratified groups"*

6:00 – 7:30 Reception and cash bar, Illini Union, Room 210.

Thursday, 7/13, morning session, 314 Altgeld Hall.

9:00 – 10:00 M. Chyba, Hawaii: *"Geometric analysis of a controlled submerged rigid body"*

10:15 – 11:15 M. Cowling, New South Wales: *"The rigidity problem for Carnot groups"*

11:30 – 12:30 D.-M. Nhieu, Georgetown: *"Instability of graphical strips and a positive answer to the Bernstein problem in the first Heisenberg group"*

12:30 Lunch

3:00/4:00 Tour of the CAVE/CUBE by Prof. George Francis, UIUC. Virtual reality environment. Come experience life in hyperbolic space! Meet in the Beckman Center lobby at 3:00 (CAVE) or 4:00 (CUBE).

Friday, 7/14, morning session, 314 Altgeld Hall.

9:00 – 10:00 A. Bloch, Michigan: *"Geometry of integrable systems and optimal control"*

10:15 – 11:15 B. Kleiner, Yale: *"Bi-Lipschitz embedding in Banach spaces"*

11:30 – 12:30 D. Herron, Cincinnati: *"Metric space inversions and quasihyperbolic geometry"*

12:30 – 2:30 Lunch

Friday, 7/14, afternoon session, 245 Altgeld Hall.

2:30 – 3:00 X. Xie, Cincinnati: *"Uniform domains and Gromov hyperbolic spaces"*

3:10 – 3:40 I. Prause, Helsinki: *"Improved quasiconformal dimension distortion on the line"*

3:40 – 4:00 Coffee break

4:00 – 4:30 H. Hakobyan, Stony Brook: *"Hausdorff dimension and quasisymmetric maps"*

7:00 – 9:00 Dinner, location TBA.

Saturday, 7/15, morning session, 245 Altgeld Hall.

9:00 – 10:00 G. Citti, Bologna: *"The Mumford–Shah functional in the sub-Riemannian structure of the visual cortex"*

10:15 – 11:15 S. Esedoglu, Michigan: *"Finding global minimizers of segmentation and denoising functionals"*

11:30 – 12:30 R. Hladky, Dartmouth: *"Variation of horizontal perimeter measure"*

12:30 – 2:30 Lunch

Saturday, 7/15, afternoon session, 245 Altgeld Hall.

2:30 – 3:00 S. Vodop'yanov, Novosibirsk: *"Differentiability of mappings in the geometry of Carnot-Caratheodory spaces and applications"*

3:10 – 3:40 M. Karmanova, Novosibirsk: *"Rectifiable sets and coarea formula for metric-valued mappings"*

3:40 – 4:00 Coffee break

4:00 – 4:30 B. Warhurst, New South Wales: *"Rigidity of free Carnot groups"*

4:40 – 5:10 L. Kovalev, Texas A&M: *"A geometric approach to accretivity"*

Abstracts

Anthony Bloch, University of Michigan, *"Geometry of integrable systems and optimal control"*

In this talk I will discuss the geometry of a number of integrable systems of interest and their relationship to optimal control problems. In particular I will discuss the dynamics of the n-dimensional smooth rigid body problem and its discretization as well as the sub-Riemmanian rigid body problem. I will also discuss geodesic flows on Stiefel manifolds and their relationship to optimal control problems, the rigid body problem and the Jacobi flow on an ellipsoid. Finally I will describe an interesting class of integrable flows on the symplectic group.

Mario Bonk, University of Michigan, *"Metrics on the boundary of negatively curved spaces"*

The Carnot-Carathéodory metric on the Heisenberg group arises in a natural way as a boundary metric of the complex unit ball in $\mathbb{C}^2$ equipped with the Bergman metric. More generally, there is a relation between metrics on the boundary of a strictly pseudoconvex domain and biholomorphically invariant metrics inside the domain. This can be used to study the boundary behavior of proper holomorphic maps in several complex variables, for example.

Roger Brockett, Harvard University, *"Some degenerate diffusion processes and random matrices associated with sub-Riemannian geometry"*

Sub-Riemannian geometry has given rise to a rich web of interconnected ideas. In this talk we will discuss results related to the higher dimensional "area processes" associated with an important special type of degenerate diffusions. Using the Brownian bridge we will compute statistical properties of the vector area process and relate them to fundamental solutions of certain prototype degenerate diffusions. Along the way we encounter a class of random matrices whose eigenvalue distributions will be explored and related to the geodesics of a particular sub-Riemannian space.

Monique Chyba, University of Hawaii, *"Geometric analysis of a controlled submerged rigid body"*

Motivated by the recent trend to build autonomous underwater vehicles, we study the geometric properties related to the motion of a controlled submerged rigid body. Such system belong to the class of simple mechanical systems with potential and uncontrolled dissipative forces. After introducing

the equations of motion, the talk will focus on the relation between the notion of decoupling vector fields for a left-invariant affine connection control system on the Lie group SE(3) and the notion of singular extremal in the time minimal problem for this same system.

Giovanna Citti, University of Bologna, *"The Mumford–Shah functional in the sub-Riemannian structure of the visual cortex"*

Abstract: We will show that the structure of the visual cortex is of sub-Riemannian type. In this structure the discrete visual signal, constituted by independent stimula, is grouped and the object reconstructed. This functionality is modelled as difference equation, which depends on the the metric of the space, and which can be interpreted as the Euler–Lagrange equation of a discrete operator. This operator Γ-converges, as the dimension of the grid tends to zero to the Mumford–Shah functional in the sub-Riemannian structure.

Michael Cowling, University of New South Wales, *"The rigidity problem for Carnot groups"*

While some Carnot groups, e.g., the Heisenberg groups and, more generally, those arising from jet spaces) have infinite-dimensional spaces of smooth contact mappings, in others, the so-called rigid groups, contact mappings are automatically real analytic, and even the spaces of contact mappings defined on small open sets are finite-dimensional. In extreme cases, all contact mappings are generated by translations and dilations. This talk surveys the problem of determining whether Carnot groups are rigid or not, and highlights recent work of Rupert McCallum on this problem.

Selim Esedoglu, University of Michigan, *"Finding global minimizers of segmentation and denoising functionals"*

Segmentation is a fundamental procedure in computer vision. It forms an important preliminary step whenever useful information is to be extracted from images automatically. Given an image depicing a scene with several objects in it, its goal is to determine which regions of the image contain distinct objects.

Variational segmentation models, such as the Mumford-Shah functional and its variants, pose segmentation as finding the minimizer of an energy. The resulting optimization problrmes are often non-convex, and may have local minima that are not global minima, complicating their solution. We

show that certain shape optimization problems that arise as simplified versions of these segmentation models can be given equivalent convex formulations, allowing us to find global minimizezrs of these non-convex problems via convex minimization techniques. In particular, we will show that a recent convex duality based algorithm due to A. Chamboile, which was originally developed for a total variation based image denoising model, can be adapted to the segmentation problem.

Hrant Hakobyan, State University of New York at Stony Brook, *"Hausdorff dimension and quasisymmetric maps"*

A subset of a line is said to be quasisymmetrically (qs) thick if every image under a qs self map of a line has positive length. Conformal dimension of a metric space is the infimal Hausdorff dimension of all its qs images. Bishop and Tyson asked if there are sets on a line which are not qs thick but still have conformal dimension one. We answer this affirmatively by showing that regular Cantor sets (in the sense of Staples and Ward) are minimal for conformal dimension if they have Hausdorff dimension one.

David Herron, University of Cincinnati, *"Metric space inversions and quasihyperbolic geometry"* (joint work with Stephen M. Buckley and Xiangdong Xie)

The class of Euclidean spatial conformal mappings is generated by reflections across planes and inversions about spheres. We define a notion of inversion valid in the general metric space setting and establish several basic facts concerning these. For example, they are quasimoebius homeomorphisms and quasihyperbolically bilipshitz. Just as in the Euclidean setting, sphericalization can be realized as a special case of inversion. In a certain sense, inversion and sphericalization are dual processes; this is a consequence of the fact that the natural identity maps associated with each of the following processes are bilipschitz: inversion followed by inversion, inversion followed by sphericalization, sphericalization followed by inversion.

We demonstrate that both inversion and sphericalization preserve the class of uniform subspaces.

We introduce a notion called annular quasiconvexity and demonstrate that a space is quasiconvex and annular quasiconvex if and only if its inversions and its sphericalizations also enjoy these properties. Moreover, in the presence of an annular quasiconvex ambient space, we obtain improved quantitative information describing how uniformity constants change.

Examples of metric spaces that are quasiconvex and annular quasiconvex include Banach spaces and upper regular Loewner spaces; the latter includes Carnot groups and certain Riemannian manifolds with non-negative Ricci

curvature. Korte has recently verified that doubling metric measure spaces which support a $(1, p)$-Poincare inequality with sufficiently small p are annular quasiconvex.

Rob Hladky, Dartmouth College, *"Variation of horizontal perimeter measure"*

I'll describe tools and techniques for studying the geometry of C^2 surfaces in general sub-Riemannian manifolds and provide explicit first and second variation formulas for the horizontal perimeter measure.

Islam Hussein, Worcester Polytechnic Institute, *"Optimal control for second-order systems on Riemannian manifolds"*

In this talk I will present some recent results on the geometry of optimal control problems for second order under-actuated dynamical systems evolving on Riemannian manifolds. I will first describe the general setting, including the dynamic interpolation problem. I will then briefly discuss the special case when the manifold under consideration has a Lie group structure, including optimal trajectory tracking for rigid body motions. Finally, I will present some recent results on the control of nonholonomic underactuated mechanical systems with some examples.

Maria Karmanova, Novosibirsk State University, *"Rectifiable sets and coarea formula for metric-valued mappings"*

We study Lipschitz mappings defined on an H^n-rectifiable metric space with values in an arbitrary metric space. We find necessary and sufficient conditions on the image and the preimage of a mapping for the validity of the coarea formula. As a consequence, we prove the coarea formula for some classes of mappings with H^k-σ-finite image. We also obtain the metric analog of the Implicit Function Theorem. All these results are extended to large classes of mappings with values in a metric space, including Sobolev mappings and BV-mappings.

Bruce Kleiner, Yale University, *"Bi-Lipschitz embedding in Banach spaces"* (joint work with Jeff Cheeger)

Motivated by a conjecture from computer science, I will discuss bi-Lipschitz embeddings $X \to V$ where X is a metric space and V is a Banach space. The focus will be on the case when X is a metric measure space satisfying a Poincare inequality. Of particular interest is the case when the target

Banach space is L^1, in which case there is a new link between embedding questions and the structure of sets of finite perimeter in X. By exploiting recent work on geometric measure theory in the Heisenberg group, we show that the Heisenberg group cannot be biLipschitz embedded in L^1, confirming a conjecture of Assaf Naor.

Leonid Kovalev, Texas A&M University, *"A geometric approach to accretivity"*

I will describe a connection between generalized accretive operators introduced by Felix Browder and the theory of quasisymmetric mappings in Banach spaces pioneered by Jussi Vaisala. The interplay of the two fields allows for geometric proofs of continuity, differentiability, and surjectivity of generalized accretive operators.

Valentino Magnani, University of Pisa, *"Intrinsic blow-up of submanifolds in stratified groups"*

Let us fix a point of a submanifold embedded in a stratified group. We will present the relationship between the position of the tangent space with respect to the grading of the group and the behaviour of the rescaled submanifold at this point, when its magnification is arbitrarily large.

Duy-Minh Nhieu, Georgetown University, *"Instability of graphical strips and a positive answer to the Bernstein problem in the first Heisenberg group"* (joint work with Donatella Danielli, Nicola Garofalo and Scott Pauls)

We study, in the first Heisenberg group $\mathbb{H}^1$, the structure of C^2 minimal graphs with empty characteristic locus and which, on every compact set, minimize the horizontal perimeter. As a corollary of our results we obtain a positive answer to a sub-Riemannian analogue of the Bernstein problem.

Istvan Prause, University of Helsinki, *"Improved quasiconformal dimension distortion on the line"*

The general dimension distortion result says that a one dimensional set goes to a set of dimension at least $1 - k$ under a planar k-quasiconformal mapping ($0 \leq k < 1$). An improved version for rectifiable sets appears in recent work of Astala, Clop, Mateu, Orobitg and Uriarte-Tuero in connection with quasiregular removability problems. We present an alternative proof of their result based on the original area distortion argument of Astala, establishing a bound of the form $1 - ck^2$, provided that either the initial or

the target set lies on a line. Connections to improved Painlevé removability for quasiregular mappings are discussed.

Christina Selby, Purdue University, *"ϵ-approximation technique for determining geometric quantities for hypersurfaces in Carnot groups of step two"*

In a Carnot group of step two the Riemannian metric is considered which makes $\{X_1, \dots, X_m, T_{1,\epsilon}, \dots, T_{k,\epsilon}\}$ an orthonormal basis, where $T_{s,\epsilon} = \sqrt{\epsilon} T_s$. Results from Riemannian geometry are then applied to obtain results in sub-Riemannian geometry such as integration-by-parts formulas, the first variation formula for the H-perimeter measure, and a second variation formula for H-perimeter measure with respect to variations in the horizontal normal direction.

Raul Serapioni, University of Trento, *"Intrinsic Lipschitz graphs in Heisenberg groups"*

We suggest a notion of Lipschitz function from subgroups of a given Heisenberg group. We investigate some of their properties, the properties of their graphs, and the relation they have with the boundary of finite perimeter sets in $\mathbb{H}^n$.

Davide Vittone, Scuola Normale Superiore, University of Pisa, *"The Bernstein problem for intrinsic graphs in Heisenberg groups"*

We will study the Bernstein problem for entire regular intrinsic minimal graphs in the Heisenberg group $\mathbb{H}^n$. In particular we will positively answer to the Bernstein problem in the case $n = 1$ and we will provide counterexamples when $n \geq 5$.

Sergey Vodop'yanov, Sobolev Institute of Mathematics, *"Differentiability of mappings in the geometry of Carnot-Caratheodory spaces and applications"*

We study the differentiability of mappings in the geometry of Carnot-Caratheodory spaces under C^1-smoothness of vector fields. We introduce a concept of *hc*-differentiability and prove the *hc*-differentiability of Lipschitz mappings of Carnot-Caratheodory spaces (a generalization of the Rademacher theorem) and a generalization of the Stepanov theorem. We establish adequate geometric and analytic properties for their proofs in particular, we prove *hc*-differentiability of rectifiable curves. Besides of this, we give a new proof of a functorial property of a correspondence "a local basis

$\mapsto$ the nilponent tangent cone", the *hc*-differentiability of a composition of *hc*-differentiable mappings and others. As a consequence, we obtain some applications to geometric measure theory on Carnot-Caratheodory spaces.

Ben Warhurst, University of New South Wales, *"Rigidity of free Carnot groups"*

We will show that free Carnot groups of step greater or equal to 4 are rigid using Tanaka prolongation.

Xiangdong Xie, University of Cincinnati, *"Uniform domains and Gromov hyperbolic spaces"* (joint work with David Herron and Nageswari Shanmugalingam)

We characterize uniform domains (in metric spaces with annular quasiconvexity) among Gromov hyperbolic domains, in terms of the quasiconformal structure on the Gromov boundary. This generalizes a result of Bonk, Heinonen and Koskela.

List of participants

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