# A RELAXATION OF OPTIMALITY FOR THE TGA

We begin by recalling the Thresholding Greedy Algorithm (TGA) introduced by Konyagin and Temlyakov in 1999. The TGA optimality is described by the notion of greedy and almost greedy bases. A basis $(e_n)_{n=1}^{\infty}$ of a Banach space $X$ (over a field $\mathbb{F}$) is said to be greedy if there exists a constant $\mathbf{C}\geqslant 1$ such that

$$\|x-G_m(x)\| \;\leqslant\; \mathbf{C}\inf_{\substack{|A|\leqslant m\\ (a_n)_{n\in A}\subset\mathbb{F}}}\left\|x-\sum_{n\in A}a_ne_n\right\|.$$

Here, $G_m(x)$ is the so-called greedy sum of $x$ of size $m$. The definition of almost greedy bases replaces the arbitrary linear combinations on the right by projections. We present properties of both greedy and almost bases as well as their characterizations.

Extending classical results, we define ($f$, greedy) bases to satisfy the condition: there exists a constant $\mathbf{C}\geqslant 1$ such that

$$\|x-G_m(x)\| \;\leqslant\; \mathbf{C}\inf_{\substack{|A|\leqslant f(m)\\ (a_n)_{n\in A}\subset\mathbb{F}}}\left\|x-\sum_{n\in A}a_ne_n\right\|,$$

where $f$ belongs to $\mathcal{F}$, a collection that contains functions like $f(x)=cx^{\gamma}$ for $c,\gamma\in[0,1]$. The definition of ($f$, almost greedy) is modified accordingly. We give characterizations of these bases, which help establish the surprising equivalence: if $f$ is a non-identity function in $\mathcal{F}$, then a basis is ($f$, greedy) if and only if it is ($f$, almost greedy). We show that ($f$, greedy) bases form a much wider class as there exist examples of classical bases that are not almost greedy but is ($f$, greedy) for some $f\in\mathcal{F}$.