Evaluate
\[ I = \int_{|z+2|=2} \frac{dz}{(z^3+1)^2} \]
using the Cauchy integral formula for derivatives.

\[ z^3 + 1 = (z+1) (z^2 - z + 1) \]
\[ z^3 + 1 = 0 \iff z = \sqrt[3]{-1} = -1, z_1, z_2 \]
\[ z_{1,2} = \frac{1 \pm i\sqrt{3}}{2} \text{ outside } \gamma \]
\[ \gamma = \{ z : |z+2i| = 2 \} \]

By the Cauchy formula for derivatives,
\[ I = \int_{\gamma} \frac{dz}{(z^3+1)^2} = \int_{\gamma} \frac{f(z)}{(z+1)^2} = \frac{2\pi i f'(-1)}{1!} \]

Here \( f(z) = \frac{1}{(z^2 - z + 1)^2} \) is analytic
inside and on \( \gamma \).

\[ f'(z) = -2 (z^2 - z + 1)^{-3}(2z-1) \]
\[ f'(-1) = \frac{2}{9} \]
\[ I = 2\pi i f'(-1) = \frac{4\pi i}{9} \]