1. (5 pts) Let $A \in M_{3 \times 3}(\mathbb{R})$. Let $\mathbb{R}^3$ have a basis $v_1, v_2, v_3$ such that $Av_1 = v_2, Av_2 = v_3, Av_3 = v_1$. Is $A$ diagonalizable over $\mathbb{R}$? Is $A$ diagonalizable over $\mathbb{C}$?
2. (5 pts) The eigenvalues of the matrix

\[ A = \begin{pmatrix} 11 & -4 & 4 \\ 14 & -4 & 7 \\ 2 & -1 & 4 \end{pmatrix} \]

are 5 and 3.

(a) (3 pts) Find bases of the eigenspaces of \( A \).

(b) (1 pt) Is the matrix \( A \) diagonalizable? Is so, then find a diagonal matrix \( D \) and an invertible matrix \( Q \) such that \( Q^{-1}AQ = D \).

(c) (1 pt) Verify that \( AQ = QD \).
3. (5 pts) Let $T: V \to V$ be a linear transformation of a real vector space $V$. Suppose $T^2 = I_V$. Prove $T$ is diagonalizable. (Hint. Observe that $\frac{v + T(v)}{2}$ and $\frac{v - T(v)}{2}$ are eigenvectors whose sum is equal to $v$.)
4. (5 pts) In a real inner product space, when the Cauchy-Schwarz inequality becomes an equality? State and prove.
5. (5 pts) Let \( W = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\} \) be a subspace of \( \mathbb{R}^4 \) with the standard inner product. Find orthonormal bases for \( W \) and \( W^\perp \).
6. (5 pts) Let \( A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{pmatrix} \) and \( b = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \end{pmatrix} \).

(a) (3 pts) Find the least squares solution of the system \( Ax = b \), that is, find \( x \in \mathbb{R}^2 \) so that \( \|Ax - b\| \) is minimum.

(b) (2 pts) Find the orthogonal projection \( p \) of \( b \) onto \( \text{Col}A \) and verify that \( b - p \) is orthogonal to \( \text{Col}A \).
7. (5 pts) Let $V$ be a real inner product space and let $T : V \to V$ be a self-adjoint operator. Let $v_1, v_2 \in V$ be eigenvectors of $T$ with different eigenvalues. Prove that $v_1$ and $v_2$ are orthogonal.