Math 231 - Merit - Practice Midterm 3

Multiple choice. Mark answers clearly on problems.

1. Suppose that the power series $\sum_{n=0}^{\infty} c_n(x-8)^n$ converges when $x = 12$ and diverges when $x = 3$. Which of the following are true statements about the convergence / divergence of this power series? Circle all correct answers.

(A) The series converges when $x = 0$.
(B) The series converges when $x = 4$.
(C) We cannot tell if the series converges or diverges for $x = 13$.
(D) The series converges when $x = 8$.
(E) The series converges when $x = 9$.

2. Which of the following series are alternating series? Circle all correct answers.

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n}$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$

(D) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\cos(\pi n)}{n}$

Consider the statements:

(A) The series diverges by the ratio test.
(B) The series converges by the ratio test.
(C) The ratio test is inconclusive.
(D) The series converges absolutely.
(E) The series converges conditionally.
(F) The series converges by the alternating series test.

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3. (5 points) Which of the above statements hold for the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^5}{n^6+n+1}$?

4. (5 points) Which of the above statements hold for the series $\sum_{n=0}^{\infty} \frac{(-3)^{n+1}}{(n+1)!}$?

Free response. Show work and circle answers.

5. Find the Taylor series for $f(x) = \cos x$ centered at $\dfrac{3\pi}{2}$.

6. Suppose that a Taylor series of a function $f(x)$ is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n^2}{5n!} x^{2n}$$

(A) Evaluate $f^{(12)}(0)$

(B) Evaluate $f^{(13)}(0)$

7. (A) Use the binomial series to find the Maclaurin series for $g(x) = \dfrac{1}{\sqrt{1-4x}}$. Do not yet simplify the binomial coefficients that arise.

(B) Write out the first three non-zero terms in this series, and simplify your coefficients as much as possible.

8. Find $\lim\limits_{x\to 0} \dfrac{1-\cos x}{1+x-e^x}$ using Maclaurin series.

9. (A) Write the Maclaurin series for $h(x) = xe^{-x^3}$.

(B) Write a series for $\int_0^1 xe^{-x^3}\,dx$ and estimate the sum to within $\frac{1}{1000}$