Worksheet #22

Math 231 AD1

Parametric Equations

1. Consider the functions $x(t) = e^t$ and $y(t) = t + 3$.
   (a) Eliminate a parameter to find a Cartesian equation of the curve.
   (b) Sketch the curve and indicate with an arrow the direction the curve is traced.
2. Consider the functions $x(t) = e^{-t}$ and $y(t) = 3 - t$.
   (a) Eliminate a parameter to find a Cartesian equation of the curve.
   (b) Sketch the curve and indicate with an arrow the direction the curve is traced.
3. Consider the functions $x(t) = 2t^2$ and $y(t) = t^3 - 3t$.
   (a) On the board, draw a Cartesian coordinate system (x- and y-axes).
   (b) Find all x-intercepts and y-intercepts. Plot them on the board, indicating the corresponding t-values.
   (c) Find all horizontal and vertical tangent lines. Plot them on the board, indicating the corresponding t-values.
   (d) Determine the x- and y-asymptotes as $t \to \pm\infty$.
   (e) Find where $x(t)$ is increasing/decreasing. Do the same for $y(t)$.
   (f) Use the information you've gathered to sketch the parametric curve.
4. Consider the functions $x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$.
   (a) Make a sketch of the path for $t \geq 0$.
   (b) Determine where there are horizontal and/or vertical tangent lines.
   (c) Find the arc length for this path on for $0 \leq t \leq 1$.
   (d) Setup an integral for the area over the x-axis and under the path for $0 \leq t \leq \pi$.
   (e) Sketch a graph of this area.
5. Consider the functions $x(t) = a\cos(t)$ and $y(t) = b\sin(t)$.
   (a) Show that these equations models a particle traveling counter-clockwise about an ellipse at one revolution per 2π units of time, starting at the point $(a, 0)$.
   (b) Determine for what t the tangent line has slope $\frac{a}{b}$.
   (c) Determine the area of an ellipse using this set of parametric equations
   (d) Write the integral for the circumference of the ellipse using this set of equations.
   (e) This is an "elliptical" integral and does NOT have a nice anti-derivative. Despite this, the formula for the circumference of an ellipse is very simple. Any guesses? (Your answer should be in terms of a and b.)

Cummulative Review I

6. State the formula for Integration by Parts.

7. For what values of p does $\int_0^1 \frac{1}{x^p}\,dx$ converge/diverge?

8. Evaluate three of the following integrals.

   (a) $\int_0^1 \arctan(x)\,dx$ (b) $\int \sin^2(x)\cos^3(x)\,dx$ (c) $\int \sec^2(x)\tan^4(x)\,dx$

   (d) $\int \sec^2(x)\tan^4(x)\,dx$ (e) $\int \frac{1}{\sqrt{x^2+4x}}\,dx$

9. Give the Partial Fraction Decompositoin of $\frac{x^3-4x-10}{x^2-x-6}$.