Worksheet #19

Math 231 AD1

New notation!

Define $\binom{k}{n} = \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} = \frac{k}{n}\cdot\frac{k-1}{n-1}\cdot\frac{k-2}{n-2}\cdots\frac{k-n+1}{1}$. Always $\binom{k}{0} = 0$

Warm-up

1. Compute $\binom{\frac{3}{2}}{1}, \binom{\frac{3}{2}}{2}, \binom{\frac{3}{2}}{3}, \binom{\frac{3}{2}}{4}$.

More Taylor Series

2. Find the Taylor series for $f(x) = e^{3x}$ centered at $x = 2$. (Hint: Draw a table.)
3. Find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$.
4. Use power series expansions to compute $\lim_{x\to 0} \frac{\sin(x)}{x}$.
5. Another Maclaurin series that you must know! $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$.
   (a) Write down the Maclaurin series for $\sqrt{1+x}$ in summation form.
   (b) Write out the first 4 terms and simplify the coefficients.
   (c) Write down an alternating series that converges to $\sqrt{17}$.
   Hint: $\sqrt{17} = \sqrt{16+1} = 4\sqrt{1+\frac{1}{16}}$
6. Here's a useful piece of information: In the notation for the Taylor polynomial, $T_n(x)$, the n is the *degree* of the polynomial, which is the highest power on x, NOT the number of terms.
   (a) Using the Taylor series from #2, write down $T_3(x)$.
   (b) What is the max error of using $T_3(x)$ to approximate e^3?