Worksheet #18

Math 231 AD1

Problems 6–8 on Worksheet 17 (posted on my website) are really good practice. Not many groups had time to work on them, so I strongly recommend completing them on your own.

Warm-up

1. Write down the Maclaurin Series (summation form AND first 3 terms) for each of the following functions. This table is REALLY IMPORTANT! (Ok?)

   (a) $\dfrac{1}{1-x}$

   (b) e^x

   (c) $\sin(x)$

   (d) $\cos(x)$

   (e) $\ln(1+x)$

   (f) $\arctan(x)$

Power Series

2. Complete the following table with $f(x) = \cos(x)$ and $a = \pi/2$, and then write down the Taylor series for $\cos(x)$ centered at $x = \pi/2$:

n	0	1	2	3	4
$f^{(n)}(x)$					
$\frac{f^{(n)}(a)}{n!}$					

   NOTE: If you were to complete this table for $a = 0$, the series you get should match the Maclaurin series for $\cos(x)$ above.

3. Evaluate the indefinite integral $\displaystyle\int \frac{e^x - 1}{x}\,dx$ as an infinite series.

4. The following power series are representations of certain functions. What are the functions?

   (a) $1 - x^{10} + x^{20} - x^{30} + \cdots$

   (b) $x^2 + x^5 + x^8 + x^{11} + \cdots$

   (c) $1 + 2x + \dfrac{4x^2}{2} + \dfrac{8x^3}{6} + \cdots$

   (d) $x^2 - \dfrac{1}{2}x^4 + \dfrac{1}{24}x^6 - \cdots$

   (e) $-\dfrac{x^2}{2^2 2!} + \dfrac{x^4}{2^4 4!} - \dfrac{x^6}{2^6 6!} + \cdots + \dfrac{(-1)^n x^{2n}}{2^{2n}(2n)!} + \ldots$

5. Use power series expansion to compute these following limit. (Hint: write out terms.)

   (a) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x}$

   (b) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

   (c) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$