Worksheet #13

Math 231 AD1

It's a TRAP!

- The Divergence Test (also called the nth Term Test) will **NOT** tell you if a series converges. NEVER say "Converges by the Divergence Test".
- Geometric series are probably the easiest kind we encounter. PLEASE check the ratio between multiple pairs of terms. Too many times, I've seen someone say "$r = \frac{1}{3}$" when that is only the ratio between the first two terms, and other terms are not geometric.
- For the Integral Test, you **MUST** check that the function (of x, NOT n) is continuous, positive, and decreasing. To show decreasing, check that the derivative is always negative (or negative once x is big enough).
- Attendance has been an issue. If you're sick, I need to know about it (with proof).
- The exam is 10 days from today. We'll have a mock exam on Friday (Mar 13), a review session on Monday (Mar 16), and no class on Wednesday (Mar 18).

Warm-up

1. This problem should take 10 seconds: For which p-values does $\int_1^\infty \frac{1}{x^p}dx$ converge?
2. State the Integral Test (if you need to look this up, please do so).
3. The next "seriously don't forget this" concerns the **Harmonic Series**: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Does this converge or diverge? What is surprising or special about it?
4. State the Comparison Theorem for integrals (from Section 7.8).

Integral Test

5. Consider a convergent series $s = \sum_{k=1}^\infty a_k$. Previously, we considered the partial sum $s_n = \sum_{k=1}^n a_k$. *Now* we also want to think about the "remainder" $R_n = \sum_{k=n+1}^\infty a_k$; stated another way, $R_n = s - s_n$, and it can be thought of as "error".
   If $f(x)$ is continuous, positive, decreasing and agrees with a_k at all integers k, then $\int_{n+1}^\infty f(x)\,dx \le R_n \le \int_n^\infty f(x)\,dx$. Use this to obtain the following approximations:
   (a) To approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ to within 10^{-5}, how many terms do you need to sum?
   (b) To approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within 10^{-5}, how many terms do you need to sum?

Preview (Comparison Tests)

6. State a Comparison Test for series similar to the one for integrals.
7. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n + n}$ converges.
8. Show that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$ diverges.
9. Do you expect $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - n - 1}$ to converge or diverge? What about $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n + \sqrt{n}}$?