Name: _____

Worksheet #8

Math 231 AD1

Friday's quiz will cover Section 7.8.
Starred questions are from the professor's worksheet and are of particular importance.
Please do problems 2, 4, and 6 at the blackboard or on a whiteboard.

Warm-up

1. Consider a function $f(x)$ on an interval $[a, b]$:
   (a) Using ds, give the formula for the arc length.
   (b) Using ds, give the formula for the surface area revolved around the x-axis.
   (c) What does ds represent? How can we write it in terms of dx?
   (d) Using dx, give the formula for the arc length.
   (e) Using dx, give the formula for the surface area revolved around the x-axis.

Arc Length and Surface Area

2. *Sketch the curve $y = 3 - 3x$ on $[0, 1]$ and use geometry to compute its length.
   (a) Use integration to compute its length.
   (b) Find the surface area for the curve when it is rotated about the x-axis.
   (c) Find the surface area for the curve when it is rotated about the y-axis.
3. *Find the length of $y = 1 + 6x^{3/2}$ on $[0, 1]$.
4. *Find the length of $y = \frac{x^3}{3} + \frac{1}{4x}$ on $[1, 2]$.
5. *Find the surface area of $y = x^3$ on $[0, 2]$ when revolved about the x-axis.
6. *Find the surface area of $y = 1 - x^2$ on $[0, 1]$ when revolved about the y-axis.

Preview (Sequences)

A *sequence* is a list of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$. A sequence behaves like a function in that you can plug in an n-value and get out the value a_n, but whereas functions are *continuous*, sequences are *discrete* (i.e. there is no $a_{2.5}$ or a_π).
Examples of sequences:
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ $\quad 1, -1, 1, -1, 1, -1, \ldots$ $\quad \sin(1), \sin(4), \sin(9), \sin(16), \ldots$

7. Let $a_n = \frac{2}{3n+1}$ and $f(x) = \frac{2}{3x+1}$. Compute $\lim_{n\to\infty} a_n$ and $\lim_{x\to\infty} f(x)$.
8. Let $a_n = \sin(2\pi n)$ and $f(x) = \sin(2\pi x)$. Compute $\lim_{n\to\infty} a_n$ and $\lim_{x\to\infty} f(x)$.
9. Just like limits of functions, sequences can converge (to some limit) or diverge. Determine convergence/divergence for the following sequences:
   $a_n = \frac{\sin(2n)}{1+\sqrt{n}}$ $\quad b_n = \frac{n^3}{n^2-1}$ $\quad c_n = \ln(n+1) - \ln(n)$.
10. Find a formula for the general term a_n from the sequence $-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots$.