Name: ____

Weirdness at Infinity

Math 231 AD1

Warm-up

1. Think *way* back to how we first computed derivatives, and then how we first computed integrals. What commonalities can you think of?

Arc Length

2. What parts of these approaches could we use to determine the *length* of a curve on some interval?
3. For the length of the curve $f(x)$ on the interval $[a, b]$, we have the formula $s = \int_a^b ds$.
   I think of s as the length of a string needed to trace the curve on that interval. Just like dx is a "little bit of x", I think of ds as a "little bit of string s".
   What is a formula for ds in terms of other "little bits of ___"?
4. Write an integral with respect to x (instead of ds) for the arc length of $f(x)$ on $[a, b]$.

Surface Area

5. Using the same methods as we did for derivatives, integrals, and now arc length, how should we go about finding surface area of $f(x)$ on $[a, b]$ revolved around the x-axis?
6. For one "slice", how much area is contributed? (Hint: this builds from earlier)
7. Write an integral for the surface area of $f(x)$ on $[a, b]$ revolved about the x-axis.

Gabriel's Horn

8. Consider the solid obtained by revolving $y = \frac{1}{x}$ about the x-axis on $[1, \infty)$. Last time, we determined its volume to be π.
9. Write an integral for its surface area. (Do not solve it yet!)
10. Use to the Comparison Test to show that it diverges.
11. Note that it has *finite* volume, but *infinite* area. This should seem *weird*!
    What are your thoughts on making sense of this strange fact?