Name: ______________________________

Worksheet #5

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance. Please do problems 2, 4, and 6 at the blackboard or on a whiteboard.

Warm-up

1. You *should* remember the definition of the integral: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$.
   (a) What is Δx?
   (b) We can choose x_i in several ways. What is x_i for
   Right endpoints Left endpoints Midpoints?
   (c) How do you find the area of a trapezoid?
2. Use long division; write $\dfrac{x^4 - 2x^3 + 3x^2 - 4x}{(x-1)^2(x^2+1)}$ as a polynomial with a rational remainder.

Review (Partial Fractions)

3. Give the partial fraction decomposition of $\dfrac{x^4+1}{x^2(x^2-x+1)(x^2+2)^2}$.
4. I'll give you the partial fraction decomposition: $\dfrac{x^2-2x-1}{(x-1)^2(x^2+1)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{Cx+D}{x^2+1}$.
   Use this to integrate $\displaystyle\int \frac{x^4 - 2x^3 + 3x^2 - 4x}{(x-1)^2(x^2+1)}\,dx$.

Approximate Integration

If we drop the limit from #1, then $\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i)\Delta x$ (an approximation).

The larger n gets, the better our approximation. Often, we want to know how large n must be to *guarantee* a certain amount of accuracy. The way we choose x_i can change the error:

5. *Let $R_n f$ be the Right-hand Approximation to $\int_a^b f(x)\,dx$ over n equal partitions.
   (a) Give a formula for $R_2 f, R_3 f$, and $R_n f$.
   (b) Suppose $|f'(x)| \le K_1$ for all x in $[a,b]$. Why is $|f(x) - f(a)| \le K_1(x-a)$?
   (c) Use this to show that the error between $R_1 f$ and $\int_a^b f(x)\,dx$ is at most $\dfrac{K_1(b-a)^2}{2}$.
6. Consider the integral $\int_0^1 \cos(x^2)\,dx$. I want to approximate this within 0.001 of the actual value using the Midpoint, Trapezoid, and Simpson's Rules. How large must n be in each case?
7. *Let $M_n f$ be the Midpoint Approximation to $\int_a^b f(x)\,dx$. Here the error depends on the second derivative, denoted $f^{(2)}(x)$. If $|f^{(2)}(x)| \le K_2$ for all x in $[a,b]$, then the error of M_1 is at most $\frac{K_2(b-a)^3}{24}$. Use this to show that the error of M_n is at most $\frac{K_2(b-a)^3}{24n^2}$.