Name: _____ **Groupmates:** _____

Worksheet #3

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.

Warm-up

1. Compute $\sin(\arccos(x/5))$, $\tan(\arcsin(3x))$, and $\cos(\arctan(x/2))$.
2. Apply the identity $\sin(2x) = 2\sin(x)\cos(x)$ to $\sin(2e^x)$.
3. What are the derivatives of $\csc(x)$ and $\cot(x)$?
4. Complete the square! $x^2 + 6x + 1$. More generally, $ax^2 + bx + c$.

Review (Trig integrals)

5. *Solve $\displaystyle\int_0^{\pi/4} \sin^2(x)\cos^2(x)\,dx$.
6. Integrals of the form $\int \sec^n(x)\tan^m(x)\,dx$:
   (a) When n is even, we use $u =$ _____.
   (b) When m is odd, we use $u =$ _____.
   (c) *When n is odd and m is even, recall $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$. Use IBP with $dv = \sec(x)\,dx$ to compute $\int \sec^3(x)\,dx$.

Trig Substitution

7. *Fill in the table

Expression	Substitution	dx	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$dx = a\cos\theta\,d\theta$	$a^2 - a^2\sin^2\theta = a^2\cos^2\theta$
$\sqrt{a^2 + x^2}$			
$\sqrt{x^2 - a^2}$			

8. When we learn a new technique, it's tempting to start applying it *everywhere*, which can lead to incorrect or more difficult applications. For the following integrals,
   1) can you solve it using a trig substitution? If so, what would that substitution be?
   2) can you see an easier way to solve it? (The answer could be "no".)
   (a) $\int x^3\sqrt{2x^2+3}\,dx$
   (b) $\int \frac{1}{2x^2+3}\,dx$
   (c) $\int \frac{x}{2x^2+3}\,dx$
   (d) $\int \frac{x^2}{2x^2+3}\,dx$
   (e) $\int \frac{e^x}{2x^2+3}\,dx$
   (f) $\int \frac{\sin(x)}{2x^2+3}\,dx$
   (g) $\int \frac{1}{(2x^2+3)^{3/2}}\,dx$
   (h) $\int \frac{1}{\sqrt{3x^2+2x+1}}\,dx$
9. *Using $x = \sin(3\theta)$, compute $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$.
10. Compute $\displaystyle\int \frac{1}{3 - 2x - x^2}\,dx$ by completing the square.