Worksheet #2 Solutions

Math 231 AD1

4a. $\int_0^1 \arctan(3x)\,dx$
Choose $u = \arctan(3x)$, $dv = dx$.
So $du = \frac{3}{1+9x^2}$ and $v = x$.
Using IBP, we get $x\arctan(3x)|_0^1 - \int_0^1 \frac{3x}{1+9x^2}\,dx$.
We can evaluate the first part: $1 \cdot \arctan(3) - 0 \cdot \arctan(0) = \arctan(3)$.
Solve the second integral using u-substitution with $u = 1 + 9x^2$.
So $du = 18x\,dx$ and $dx = \frac{du}{18x}$.
Change limits: $x = 0 \to u = 1$, $x = 1 \to u = 10$.
Now we have $\arctan(3) - \int_1^{10} \frac{du}{6u}$.
Evaluate the second integral directly to get $\arctan(3) - \frac{1}{6}\ln(u)|_1^{10}$.
Simplify: $\arctan(3) - \ln(10)/6$.

4b. $\int (\ln(x))^2\,dx$
Use IBP with $u = (\ln(x))^2$ and $dv = dx$.
Chain rule tells us $du = 2\ln(x)/x$.
Use IBP formula to get $x(\ln(x))^2 - \int 2\ln(x)\,dx$.
Use IBP on the second integral with $u = 2\ln(x)$ and $dv = dx$.
We then get $x(\ln(x))^2 - (2x\ln(x) - \int 2\,dx)$.
Finally, we get $x(\ln(x))^2 - 2x\ln(x) + 2x + C$.

5. $\int x^3 e^{x^2}\,dx$
IBP doesn't work initially with $dv = e^{x^2}\,dx$ because we have no way to integrate e^{x^2}.
Start with a substitution (I use w to avoid confusion with u and dv in IBP): $w = x^2$, so $dw = 2x\,dx$.
After rewriting and cancelling an x, we get $\int \frac{1}{2}x^2 e^w\,dw$.
But we need everything in terms of w.
Luckily, $x^2 = w$, so we get $\int \frac{1}{2}we^w\,dw$.
NOW, use IBP: $u = w$ and $dv = e^w\,dw$.
IBP gives us $we^w - \int e^w\,dw$.
Solving gives us $we^w - e^w + C$, but we need the final answer to be in terms of x.
Lastly, we have $x^2e^{x^2} - e^{x^2} + C$.

6. I recommend choosing u in IBP according to **LIATE**:
Logarithmic
Iinverse Trig
Algebraic (polynomial)
Trig
Exponential

8c. $\int \sin^2(x)\cos^3(x)\,dx$.
This is a trig integral that will use u-substitution.
Since the power on $\cos(x)$ is odd, choose $u = \sin(x)$.

Using the substitution, we get $\int u^2 \cos^2(x)\, du$.
To get rid of $\cos^2(x)$, we use the identity $\cos^2(x) = 1 - \sin^2(x) = 1 - u^2$.
Now we have $\int u^2(1 - u^2)\, du$. Multiply it out and integrate: $\int u^2 - u^4\, du = \frac{u^3}{3} - \frac{u^5}{5} + C$.
Write in terms of x to get $\frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C$.

9c. $\int \sec(x)\tan^3(x)\, dx$.
This trig integral will use u-substitution with $u = \sec(x)$ since the power on $\tan(x)$ is odd.
The substitution gives us $\int \tan^2(x)\, du$, and we to write $\tan^2(x)$ in terms of u.
Use $\tan^2(x) = \sec^2(x) - 1 = u^2 - 1$ to get $\int u^2 - 1\, du$.
Solve and write in terms of x to get $\frac{\sec^3(x)}{3} - \sec(x) + C$.