

Name: \_\_\_\_\_

## Worksheet #24

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.

**ALL** sketches of curves must be done on the board.

Suggested exercises: Section 10.1 (page 641–642), 5–18, 28.

### Parametric Equations

- \*Consider the functions  $x(t) = e^t$  and  $y(t) = t + 3$ .
  - Eliminate a parameter to find a Cartesian equation of the curve.
  - Sketch the curve and indicate with an arrow the direction the curve is traced.
- \*Consider the functions  $x(t) = e^{-t}$  and  $y(t) = 3 - t$ .
  - Eliminate a parameter to find a Cartesian equation of the curve.
  - Sketch the curve and indicate with an arrow the direction the curve is traced.
- Consider the functions  $x(t) = 2t^2$  and  $y(t) = t^3 - 3t$ .
  - On the board, draw a Cartesian coordinate system ( $x$ - and  $y$ -axes).
  - Find all  $x$ -intercepts and  $y$ -intercepts. Plot them on the board, indicating the corresponding  $t$ -values.
  - Find all horizontal and vertical tangent lines. Plot them on the board, indicating the corresponding  $t$ -values.
  - Determine the  $x$ - and  $y$ -asymptotes as  $t \rightarrow \pm\infty$ .
  - Find where  $x(t)$  is increasing/decreasing. Do the same for  $y(t)$ .
  - Use the information you've gathered to sketch the parametric curve.
- \*Consider the functions  $x(t) = e^t \cos(t)$  and  $y(t) = e^t \sin(t)$ .
  - Make a sketch of the path for  $t \geq 0$ .
  - Determine where there are horizontal and/or vertical tangent lines.
  - Find the arc length for this path on for  $0 \leq t \leq 1$ .
  - Setup an integral for the area over the  $x$ -axis and under the path for  $0 \leq t \leq \pi$ .
  - Sketch a graph of this area.
- \*Consider the functions  $x(t) = a \cos(t)$  and  $y(t) = b \sin(t)$ .
  - Show that these equations models a particle traveling counter-clockwise about an ellipse at one revolution per  $2\pi$  units of time, starting at the point  $(a, 0)$ .
  - Determine for what  $t$  the tangent line has slope  $\frac{a}{b}$ .
  - Determine the area of an ellipse using this set of parametric equations
  - Write the integral for the circumference of the ellipse using this set of equations.
  - This is an "elliptical" integral and does NOT have a nice anti-derivative. Despite this, the formula for the circumference of an ellipse is very simple. Any guesses? (Your answer should be in terms of  $a$  and  $b$ .)

### Cummulative Review I

6. State the formula for Integration by Parts.

7. For what values of  $p$  does  $\int_0^1 \frac{1}{x^p} dx$  converge/diverge?

8. Evaluate three of the following integrals.

(a)  $\int_0^1 \arctan(x) dx$       (b)  $\int \sin^2(x) \cos^3(x) dx$       (c)  $\int \sec^2(x) \tan^4(x) dx$

(d)  $\int \sec^2(x) \tan^4(x) dx$       (e)  $\int \frac{1}{\sqrt{x^2 + 4x}} dx$

9. Give the Partial Fraction Decompositoin of  $\frac{x^3 - 4x - 10}{x^2 - x - 6}$ .

10. State the error bounds  $|E_M|$ ,  $|E_T|$ , and  $|E_S|$ , including how each  $K$  is chosen.