Name: _____

Worksheet #21

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.
Suggested exercies: Section 11.10 (page 765–766), 43–46, 51–54.

Warm-up

1. State the Alternating Series Estimation Theorem.
2. State Taylor's Inequality.
3. What is the Maclaurin series for $\arctan(x)$?
4. State the Binomial series (the power series for $(1+x)^k$).

Taylor Series Estimation

5. *You have a mystery function f. You know the following about it:
$$f(1)=2, f'(1)=0, f^{(2)}(1)=5, f^{(3)}(1)=1, f^{(4)}(x)=\frac{1}{\sqrt{x}5^x}$$
Write an algebraic expression that approximates $f(2)$ to at least $\frac{1}{100}$ accuracy. Be sure to justify your answer.
6. Estimate the value of $\cos(5)$ using $T_5(x)$ for $f(x)=\cos 3x$ centered at $a=\frac{\pi}{2}$.
Use Taylor's inequality to estimate the maximum value of $|R_5|$.
7. (a) *Find the power series for $\sqrt{1+x}$.
   (b) *Find an alternating series that converges to $\sqrt{17}$. Hint: $\sqrt{17}=\sqrt{16+1}=4\sqrt{1+\frac{1}{16}}$.

MORE Taylor Series

8. Evaluate the indefinite integral $\displaystyle\int \frac{e^x-1}{x}\,dx$ as an infinite series.
9. The following power series are representations of certain functions. What are the functions?
   (a) $1-x^{10}+x^{20}-x^{30}+\cdots$
   (b) $x^2+x^5+x^8+x^{11}+\cdots$
   (c) $1+2x+\dfrac{4x^2}{2}+\dfrac{8x^3}{6}+\cdots$
   (d) $x^2-\dfrac{1}{2}x^4+\dfrac{1}{24}x^6-\cdots$
   (e) $-\dfrac{x^2}{2^2 2!}+\dfrac{x^4}{2^4 4!}-\dfrac{x^6}{2^6 6!}+\cdots+\dfrac{(-1)^n x^{2n}}{2^{2n}(2n)!}+\cdots$
10. Use power series expansions to compute the following limits.
   (a) $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x}$
   (b) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$
   (c) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}$
   (d) $\displaystyle\lim_{x\to 0}\frac{\sin(x)-x+\frac{1}{6}x^3}{x^5}$