Name: ____

Worksheet #20

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance. From Worksheet 19, Problem 4 is *very* important. Do this problem on your own; ask if you have questions. Suggested exercies: Section 11.10 (page 765), 13–20,29–38.

Warm-up

1. What is the difference between a Maclaurin series and a Taylor series?
2. State the Maclaurin series for e^x.
3. State the Maclaurin series for $\frac{1}{1-x}$.
4. Complete the following table with $f(x) = \sin(x)$ and $a = \pi/2$:

n	0	1	2	3	4
$f^{(n)}(x)$					
$\frac{f^{(n)}(a)}{n!}$					

Taylor Series

5. *(Polynomials as Power Series) A polynomial is a special case of a power series. In this problem, we think about what we can take as the center and what the radius of convergence will be. Let $f(x) = 3 + 7x + 13x^2$.
   (a) If we write $f(x) = \sum_{n=0}^{\infty} c_n x^n$. What are $c_0, c_1, c_2, c_3, \ldots, c_n, \ldots$?
   (b) Calculate $f(0), f'(0), \frac{1}{2}f''(0), \frac{1}{6}f'''(0), \ldots, \frac{1}{n!}f^{(n)}(0)$.
   What is the relation with the coefficients c_n of $f(x)$?
   (c) We can also think of $f(x)$ as a power series centered at $a = 1$, by writing $f(x) = \sum_{n=0}^{\infty} d_n (x-1)^n$. What are the coefficients d_n?
6. *Consider a power series $p(x)$ that starts out as $p(x) = 1+2x+3x^2+\cdots$. Is it possible that $p(x)$ is the Taylor series (centered at $x = 0$) of a function $f(x)$ with value $f(0) = 0$ at $x = 0$? Could it be the Taylor series for a function $g(x)$ with $g''(0) = 6$?
7. *Find the Taylor series $T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ for the following functions.
   (a) $f(x) = e^{3x}, a = 2$.
   (b) $f(x) = \frac{1}{(1-x)^2}, a = 0$.
   (c) $f(x) = \sin(x), a = \frac{\pi}{2}$.
8. *Suppose $f(x)$ has derivatives at $x = 5$ that are given by $f^{(n)}(5) = \frac{(-1)^n n!}{(n+1)3^n}$.
   (a) Find the Taylor series $T(x)$ for this function centered at $x = 5$.
   (b) Find the radius of convergence of this Taylor series.

MORE Taylor Series

9. Give the Taylor series for $f(x) = x^3$ with center $a = 2$. What geometric pattern do you notice about the coefficients?
10. Evaluate the indefinite integral $\int \frac{e^x - 1}{x}\,dx$ as an infinite series.
11. The following power series are representations of certain functions. What are the functions?
    (a) $1 - x^{10} + x^{20} - x^{30} + \cdots$
    (b) $x^2 + x^5 + x^8 + x^{11} + \cdots$
    (c) $1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \cdots$
    (d) $x^2 - \frac{1}{2}x^4 + \frac{1}{24}x^6 - \cdots$
    (e) $-\frac{x^2}{2^2 2!} + \frac{x^4}{2^4 4!} - \frac{x^6}{2^6 6!} + \cdots + \frac{(-1)^n x^{2n}}{2^{2n}(2n)!} + \ldots$
12. Use power series expansion to compute these following limit
    (a) $\lim_{x\to 0} \frac{1 - \cos x}{x}$
    (b) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$
    (c) $\lim_{x\to 0} \frac{\sin x}{x}$
    (d) $\lim_{x\to 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$