Name: _____ **Groupmates:** _____

Worksheet #2

Math 231 AD1

Each group must write up solutions to the starred (*) questions on a separate piece of paper. Only turn in one write-up per group. Each group member must sign their name to the solution, indicating that they both contributed and understand the submitted solution.

Trig Identities

1. You're all good calculus students and certainly know the important identity

$$\sin^2(x) + \cos^2(x) = 1.$$

   Many of you already know the two similar identities (one for sec and tan, the other for csc and cot), but to make life easier, we can avoid having to memorize all three. Beginning with $\sin^2(x) + \cos^2(x) = 1$, divide both sides by $\cos^2(x)$ and simplify to obtain one identity. Begin again with $\sin^2(x) + \cos^2(x) = 1$, and this time divide both sides by $\sin^2(x)$ and simplify to obtain the other.

2. What are the half-angle formulas?

$$\sin^2(x) = \qquad\qquad \cos^2(x) =$$

3. How about the double-angle formulas? (there are multiple answers for the second one)

$$\sin(2x) = \qquad\qquad \cos(2x) =$$

Integration By Parts (IBP)

4. *Evaluate the integrals $\int_0^1 \arctan(3x)\,dx$ and $\int (\ln(x))^2\,dx$.

5. *Evaluate the integral $\int x^3 e^{x^2}\,dx$ (Hint: your first step may not be IBP).

6. The following integrals require IBP. What are some ideas on how to choose u and dv? Make a choice for u and dv, and *setup* the first iteration of IBP. Do NOT evaluate the integral you get from applying IBP.

$$\int x^2 \ln(x)\,dx \qquad \int x \arcsin(x)\,dx \qquad \int x^3 \sin(x)\,dx \qquad \int e^x(2x^3+1)\,dx$$

Trig Integrals

7. Last time, we discovered that IBP is the inverse of the _____ rule for derivatives. What derivative rule can we "undo" with a u-substitution?

8. Consider the integral $\int \sin^n(x)\cos^m(x)\,dx$ where m and n are whole numbers.
   (a) What method would you use if either m or n (or both) are odd?
   (b) What method would you use if both m and n are even?
   (c) *Evaluate the integral $\int \sin^2(x)\cos^3(x)\,dx$.
   (d) Evaluate the integral $\int \sin^4(x)\cos^2(x)\,dx$.
   (e) Do the above methods change if you have an integral like $\int \sin^3(x)\,dx$ or $\int \cos^2(x)\,dx$?

9. Consider the integral $\int \sec^n(x)\tan^m(x)\,dx$ where m and n are whole numbers.
   (a) What method would you use if n is even?
   (b) What method would you use if m is odd?
   (c) What method would you use if n is odd and m is even?
   (d) *Evaluate the integral $\int \sec(x)\tan^3(x)\,dx$.
   (e) Evaluate the integral $\int \sec^4(x)\tan^2(x)\,dx$.