Name: ________________ Groupmates: ________________________________

Worksheet #1

Math 231 AD1

1. What exactly does $\int_0^1 f(x)\,dx$ represent? How does this differ from $\int f(x)\,dx$?

   "Area" Antiderivative

2. What is the Fundamental Theorem of Calculus (FTC)?

   Book

3. Evaluate the following

   (a) $\int \sin(2x)\,dx$ $u=2x$

   (b) $\int x\cos(x^2)\,dx$ $u=x^2$

   (c) $\int \frac{x}{x^2-1}\,dx$ $u=x^2-1$

   (d) $\int \frac{u\,du}{1+\sqrt{u}}$

   $v=1+u^{1/2}$

   $dv=\frac{1}{2}u^{-1/2}du \rightarrow du=\underline{2\sqrt{u}}\,dv$

   $\int \frac{u}{1+\sqrt{u}}du=\int \frac{u}{v}\cdot 2\sqrt{u}\,dv$

   $=2\int \frac{u^{3/2}}{v}dv$

   $u^{1/2}=v-1$, so $u^{3/2}=(v-1)^3$

   $=2\int \frac{(v-1)^3}{v}dv.$

4. What is wrong with the equation

$$\int_{\pi/3}^{\pi} \sec(x)\tan(x)\,dx = \sec(x)\Big|_{\pi/3}^{\pi} = -3?$$

   Check continuity.

5. What is the integration by parts formula?

6. Now we will derive the formula so you will understand why it works and won't have to memorize it!

   (a) According to the product rule, $\frac{d}{dx}f(x)g(x) =$ ____________________

   (b) Integrate both sides of your equation; simplify the left side using the FTC.

   (c) Make appropriate substitutions to rewrite your answer to look like the integration by parts formula from earlier.

   (d) One way to remember the integration by parts formula is to notice that it corresponds to *which* rule for taking derivatives?

7. What is wrong with saying $\int x^2\cos(x)dx = \frac{1}{3}x^3\sin(x)$?

8. Just like any other integration technique, the goal of integration by parts is to transform an integral that we don't know how to do into one that we can figure out. You want to keep this in mind when making choices for u and dv. In this question, I'll try to help you build intuition for what to choose.

   (a) If you differentiate an n-degree polynomial n times, what type of function will you get?

   (b) Suppose you want to integrate $f(x) = p(x)g(x)$ where $p(x)$ is a polynomial, and we know how to integrate $g(x)$ as many times as we want. For example, we could have $f(x) = (x^3 + 2)\sin(x)$ or $f(x) = x^4 e^x$. Can you use integration by parts? If so, how would you choose u and dv?

   (c) Supose $f(x) = \ln(x)g(x)$ where $g(x)$ is some other integrable function. Should you choose $\ln(x)$ to be u or dv? Why?

9. A function like e^x does not change with integration of differentiation, so if we have $f(x) = e^x g(x)$, we hope that $g(x)$ "changes" in some nice way. But what if $g(x)$ doesn't!?

   (a) Consider $\int e^x \cos(x)dx$. Done in lecture

   (b) Choose u and dv and apply integration by parts.

   (c) What can you say about your answer from the previous part?

   (d) Try applying integration by parts again. Do you see anything that's helpful?

10. (a) What is $\frac{d}{dx}(x \arcsin(x))$? What rule did you use to find this?

    (b) Solve the previous part for $\arcsin(x)$.

    (c) Apply the FTC to compute $\displaystyle\int_0^1 \arcsin(x)dx$. $= 1 - \frac{\pi}{2}$

11. By beginning with finding the derivative of $x\ln(x)$, use the method from the previous problem to compute $\int_1^e \ln(x)dx$. $= 1$