Name: ______________________________

Worksheet #17

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.

Warm-up

1. *Can the Ratio test tell you anything about $\sum_{n=1}^{\infty} \frac{1}{n^4}$?
2. (a) *Is it true that if $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} |a_n|$ is divergent also?

   (b) *If $\sum_{n=1}^{\infty} |a_n|$ is convergent, then must $\sum_{n=1}^{\infty} a_n$ also converge?

   (c) *If $\sum_{n=1}^{\infty} a_n$ is convergent, then is $\sum_{n=1}^{\infty} |a_n|$ also convergent?

Strategy for Testing Series

Definition: Let $\{a_n\}$ and $\{b_n\}$ be two sequences with $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$.
In the following problems, we say "$\{a_n\}$ *goes to zero faster than* $\{b_n\}$" if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$.

3. *Is the series $\sum_{n=1}^{\infty} \frac{n!}{n^2}$ convergent?
   Which sequence goes to zero faster: $\{1/n^2\}$ or $\{1/n!\}$?
4. *$n!$ versus 2^n: Show that $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges. What does this tell you about $\lim_{n\to\infty} \frac{2^n}{n!}$?
   Which sequence goes to zero faster: $\{1/2^n\}$ or $\{1/n!\}$?
5. *Determine whether the series is (absolutely or conditionally) convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^2+4}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n^2)!}{n^n}$ (c) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^n+3^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^4)^n}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$ (g) $\displaystyle\sum_{n=1}^{\infty} \frac{(1/3)^{2n}}{n^2+4}$ (h) $\displaystyle\sum_{n=1}^{\infty} e(\ln(2))^n$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{1+(-1)^n\sqrt{n}}{n+4}$ (j) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\pi n/2)}{n}$ (k) $\displaystyle\sum_{n=1}^{\infty} ax^n$

Power Series

6. Consider the series $\sum_{n=0}^{\infty} (x-2)^n$

   (a) *For what values of x does the series absolutely converge?

   (b) What is the value of the series (in terms of x) when it converges?

   (c) Let $f(x) = \sum_{n=0}^{\infty} (x-2)^n$. When the series converges, what is the value of $f'(x)$?
7. Consider the following series: Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$.

   (a) *For what values of x does the series absolutely converge?

   (b) Compute the first four derivatives of $f(x)$. Try to simplify $f''(x)$ and $f^{(4)}(x)$.

   (c) How are $f(x)$, $f''(x)$, and $f^{(4)}(x)$ related? What about $f'(x)$ and $f^{(3)}(x)$?

   (d) What *familiar* function do you think $f(x)$ is? What about $f'(x)$?