

Name: _____

Worksheet #13

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.

"Showing your work check": Choose two parts of #5 and # 8 to do on the board.

Suggested exercises: Section 11.4 (page 726), 3–32.

Warm-up

1. State the Comparison Test (be precise; avoid words like "bigger/smaller").
2. State the Limit Comparison Test (LCT, and again, be precise).
3. List the names of all other tests you currently know. (I can think of 5).

Comparison Tests

4. *Suppose I have three sequences satisfying $0 \leq a_n \leq b_n \leq c_n$.
 - (a) If $\sum_{n=1}^{\infty} b_n$ converges, for what series is the Comparison Test valid?
For what series is the Comparison Test invalid (and why)?
 - (b) If $\sum_{n=1}^{\infty} b_n$ diverges, for what series is the Comparison Test valid?
For what series is the Comparison Test invalid (and why)?
5. *Determine whether the series converges or diverges.

(a) $\sum_{n=2}^{\infty} \frac{n^3}{n^4 - 1}$

(d) $\sum_{n=0}^{\infty} \frac{n + 4^n}{n + 6^n}$

(g) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^2 e^{-n}$

(b) $\sum_{n=1}^{\infty} \frac{n - 1}{n^2 \sqrt{n}}$

(e) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

(h) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n^{n+1}}}$

(c) $\sum_{n=0}^{\infty} \frac{1 + \sin n}{10^n}$

(f) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Preview

7. We know that the Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. The Comparison Tests tell us that things like $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ and $\sum_{n=2}^{\infty} \frac{1}{n-\ln n}$ also diverge. Consider the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$.
 - (a) Plot the sequence terms on a graph.
 - (b) Plot the partial sums s_1, s_2, \dots, s_8 on a graph. (Using a calculator is OK)
 - (c) Do you think this series (the *Alternating Harmonic Series*) converges or diverges?
 - (d) What conditions are needed for an Alternating Series $\sum_{n=1}^{\infty} (-1)^n a_n$ to converge?
8. Use any tests you know to determine whether the series converges or diverges.
 - (a) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$
 - (c) $\sum_{n=1}^{\infty} n e^{-n^2}$
 - (e) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$
 - (b) $\sum_{n=3}^{\infty} \frac{1}{n \sqrt{\ln n}}$
 - (d) $\sum_{n=1}^{\infty} n \sin(1/n)$
 - (f) $\sum_{n=2}^{\infty} \frac{1}{n^{\ln \ln(n)}}$