Name: ______________________________

Worksheet #12

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.
Do problems 5 and 6 on the board. Suggested exercies: Section 11.3 (page 721), 9–32.

Warm-up

1. This problem should take 10 seconds: For which p-values does $\int_1^\infty \frac{1}{x^p}dx$ converge?
2. State the Integral Test (if you need to look this up, please do so).
3. The next "seriously don't forget this" concerns the **Harmonic Series**: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Does this converge or diverge? What is surprising or special about it?
4. State the Comparison Theorem for integrals (from Section 7.8).

Integral Test

5. *If applicable, use the Integral Test to determine if the following series are convergent or divergent; otherwise, state why the Integral Test can't be used.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}}$
   (b) $1 + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} + \dfrac{1}{4\sqrt{4}} + \dots$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$
   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(3n+2)^2}$
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n}$
   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$
6. Consider a convergent series $s = \sum_{k=1}^{\infty} a_k$. Previously, we considered the partial sum $s_n = \sum_{k=1}^{n} a_k$. *Now* we also want to think about the "remainder" $R_n = \sum_{k=n+1}^{\infty} a_k$; stated another way, $R_n = s - s_n$, and it can be thought of as "error".
   If $f(x)$ is continuous, positive, decreasing and agrees with a_k at all integers k, then $\int_{n+1}^{\infty} f(x)\,dx \le R_n \le \int_n^{\infty} f(x)\,dx$. Use this to obtain the following approximations:
   (a) *To approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ to within 10^{-5}, how many terms do you need to sum?
   (b) *To approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within 10^{-5}, how many terms do you need to sum?

Preview (Comparison Tests)

7. State a Comparison Test for series similar to the one for integrals.
8. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n + n}$ converges.
9. Show that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$ diverges.
10. Do you expect $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - n - 1}$ to converge or diverge? What about $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n + \sqrt{n}}$?