Name: ________________________________

Worksheet #11

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.
Do problems 5, 6a, and the sketches from 9 and 12 on the board.

Warm-up

1. *If $\sum_{n=1}^{\infty} a_n = 3$, then what can you say about $\{a_n\}$?
2. *If $\lim_{n\to\infty} a_n = \frac{1}{3}$, then what can you say about $\sum_{n=1}^{\infty} a_n$?
3. We are given the partial sum $s_n = \frac{n-1}{n+1}$; find a_n.

11.2 Series

3. I broke all of math by showing that $0 = 1$. Help me fix it. What did I do wrong?
   $0 = 0+0+\cdots = (1-1)+(1-1)+\cdots = 1+(-1+1)+(-1+1)+\cdots = 1+0+\cdots = 1$
4. Last time we used the same trick to show $\displaystyle\sum_{n=1}^{\infty} r^{n-1} = \frac{1}{1-r}$. What did we do differently?
5. Does $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2-1}$ converge or diverge?
6. *Consider the sequence $a_n = \frac{2}{n^2-1}$.
   (a) Find the partial fraction expansion of a_n.
   (b) Find a formula for the partial sum $s_n = \displaystyle\sum_{k=2}^{n} \frac{2}{k^2-1}$.
   (c) Use the formula for s_n to find $\displaystyle\sum_{k=2}^{\infty} \frac{2}{k^2-1}$.
7. *Consider the sequence $a_n = \ln(1+\frac{1}{n})$.
   (a) Find $\lim_{n\to\infty} a_n$.
   (b) Using that $\ln(1+\frac{1}{n}) = \ln(\frac{n+1}{n}) = \ln(n+1) - \ln(n)$, find a formula for s_n.
   (c) Determine $\sum_{n=1}^{\infty} a_n$.

Preview (Integral Test)

8. Seriously, don't forget this! For what p-values does $\displaystyle\int_1^{\infty} \frac{1}{x^p}dx$ converge?
9. Sketch the function $y = 1/x$ on the interval $[1, \infty)$. On the same graph, sketch the Riemann sum approximation with $\Delta x = 1$ (width of boxes) and left endpoints.
10. Write an infinite series for the area of the boxes.
11. Is this series smaller or bigger than the corresponding integral?
12. Does this series converge or diverge? Why?
13. Repeat 9–12 with the function $y = 1/x^2$, except use *right* endpoints to make the boxes.
14. For what values of p do you think $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ will converge?