Name: ________________________________

Worksheet #10

Math 231 AD1

Starred questions are from the professor's worksheet and are of particular importance.
Do problems 2, 3, and 4 on the board.

Warm-up

1. Describe $(2n)!$. Compare this to $2n!$.
2. Does the sequence $a_n = \frac{n!}{n^n}$ converge or diverge?
3. Suppose $\sum_{k=1}^{\infty} a_k = L$, and let $s_n = \sum_{k=1}^{n} a_k$.
   (a) What is $\lim_{n\to\infty} s_n$? (b) Find $\lim_{n\to\infty} s_{n+1} - s_n$. (c) What is $\lim_{n\to\infty} a_n$?
   (d) If a *series* $\sum_{n=1}^{\infty} a_n$ converges, what must the *sequence* a_n do?
   This idea is called the **Divergence Test**.

Geometric Series

4. *Show that $(1-r)(1+r+r^2+\cdots+r^{n-1}) = 1-r^n$.
5. *Let $s_n = a(1+r+r^2+\cdots+r^{n-1})$.
   (a) Obtain the result from last time: $s_n = \sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}$.
   (b) When does $\lim_{n\to\infty} s_n$ converge and what is the value?
6. *Using 5(b), determine the value of the series. (Pay close attention to the notation)
   (a) $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ (b) $\sum_{n=1}^{\infty} \frac{3}{2^n}$ (c) $\sum_{n=2}^{\infty} \frac{1}{3^{n-1}}$ (d) $\sum_{n=0}^{\infty} \frac{2}{4^n}$ (e) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^{n-1}}$

Preview (Divergence Test / Telescoping Series)

7. Find the first several partial sums $(s_1, s_2, \ldots)$, and then determine whether the series converges or diverges. (finding its value if it does converge)
   (a) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3}}{n}$
   (b) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}$
   (c) $\sum_{n=0}^{\infty} \frac{2}{n^2+3n+2}$
   (d) $\sum_{n=0}^{\infty} \frac{1}{n^2-1}$
8. Does $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge? Why?
9. How can I show that $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$ converges or that $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ diverges?