Name: ____

Mock Exam #2.2 (Worksheet #16)

Math 231 AD1

Good practice: Section 11.7 (page 740), 1–38.

8.2: Arc Length and Surface Area

When finding surface areas, understand when the radius r should be x versus $f(x)$ (similarly y or $g(y)$). Always check your limits of integration.

11.1: Sequences

The **convergence** and **divergence** of the sequence $\{a_n\}_{n=1}^{\infty}$ depends only on $\lim\limits_{n\to\infty} a_n$.

We've been working a LOT with *series*, don't be confused if you get a question just about *sequences*. A common mistake is to get $\lim\limits_{n\to\infty} a_n \neq 0$ and say that the sequence **diverges**. A sequence can **converge** to any number, but if that number is not 0, then the *series* $\sum a_n$ will diverge.

Review the many properties of sequences. Know the Monotone Convergence Theorem.

11.2: Series

If you have a ratio r for some geometric series, then **test it** on multiple pairs of terms.

The **Divergence Test** (nth-term test) is a good starting point for any series. If nothing else, whenever you look at a series, you should ask "do the terms go to 0"?

For **Telescoping Series**, find a smaller partial sum, say s_5 or s_6. From that, try to write a formula for s_n. The value of the series is given by the limit $\lim_{n\to\infty} s_n$.

11.3: Integral Test

Setup: Choose $f(x)$ to match a_n at each value of n. Use x and dx in your integral, not n.

Check initial conditions: The *function* must be **continuous** (makes sense only for $f(x)$, not for a_n), **decreasing** ($f'(x) < 0$. It's not enough for $a_n > a_{n+1}$), and **positive** ($f(x) \geq 0$). Only use the Integral Test when you know how to integrate the function.

11.4: Comparison Tests

You **must** have positive terms. This condition is being ignored far too often.

I prefer the Limit Comparison Test when I have polynomials in both the numerator and denominator. Comparison Test is good if there's a mix of polynomials & other functions (trig, log, etc.)

11.5: Alternating Series

If the series is alternating, but the non-alternating part b_n is not decreasing or positive, try other methods. AST Error is the easiest error bound we've covered. Know whether you have an under- or over-estimate. (Is the first term an over- or under-estimate? What about the second or third?)

11.6: Ratio Test

The conclusion of the **Ratio Test** can tell you **absolute converge** or **divergence**, but not **conditional convergence**. There may be multiple steps (tests) to determine the kind of convergence.

General Comments

Taking accurate limits is *crucial* to your success. Faulty algebra, incorrectly identifying highest powers in a fraction, and being sloppy and "plugging in ∞" are costly mistakes.

Leaving off the beginning terms of a series does not affect convergence or divergence. Thus, many tests for convergence of a series are valid as long as conditions are met for all n beyond a some integer N. For example, because $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 - 99.5}$ begins with several negative terms, Comparison Tests do not apply. However, all terms are positive once $n \geq 10$, so we can use Comparison Tests if we mention that only the first several terms are negative.

After the Mock Exam, work in groups to solve the following problems.

1. Explain whe the chain of inequalities $\sum_{n=1}^{11} e^{-n} \leq \int_0^{11} e^{-x}\,dx \leq \sum_{n=0}^{10} e^{-n}$ is true.
2. For what positive integers k does the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ converge?
3. True or False:
   (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.
   (b) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
   (c) All alternating series are conditionally convergent.
   (d) Every monotonic sequence is convergent.
   (e) The infinite sum $1 - \pi + \pi^2 - \pi^3 + \dots$ converges to $\frac{1}{1+\pi}$.
4. As precisely as possible, state the Alternating Series Remainder Theorem. Give an example of how to use it.
5. As precisely as possible, state the Ratio Test.
6. As precisely as possible, state the Limit Comparison Test.
7. (Multiple Choice) Suppose $a_n > 0$ for every n, and let $s_n + a_1 + a_2 + \cdots + a_n$.

   If $s_n \leq 12$ for every n, then what can be concluded about the series $\sum_{n=1}^{\infty} a_n$?
   (a) The series diverges
   (b) There is not enough information to determine whether or not hte series converges
   (c) The series converges to 0
   (d) The series converges to 12
   (e) The series converges to a positive number, but we cannot say what the number is
8. Evaluate $\lim_{t\to 0} \left(t^2 \sum_{n=0}^{\infty} (1-t^2)^n \right)$.
9. Determine **all** values of q for which the series $\sum_{n=1}^{\infty} \frac{e^{-qn}}{n^2}$ converges. Justify your answer in detail.

1. (5 points each) Determine if the series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Show your reasoning. No work, no credit.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{5}{\sqrt[3]{n^2+5n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan(n)}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

(d) $\sum_{n=2}^{\infty} \frac{\sqrt[3]{n^3 - n^6}}{n^3}$

(e) $\sum_{k=0}^{\infty} \frac{k^2 - 2k + 5}{4k^5 + 9}$

(f) $\sum_{n=1}^{\infty} (-1)^n n e^{-n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - n + 1}{\sqrt{n^5 + 300n^3 + 2}}$

(h) $\displaystyle\sum_{n=1}^{\infty} \cos^n\left(\frac{\pi}{4}\right)$

2. (6 points) Consider the sequence $\{a_n\}_{n=1}^{\infty}$ where $a_1 = 2$ and $a_n = \frac{1}{2}(a_{n-1} + 6)$. Does this sequence converge or diverge? Find the limit if it converges.

3. (6 points) Find the sum of $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{8} + \frac{1}{27} + \frac{1}{16} + \frac{1}{81} + \dots$.

4. (12 points) Consider the curve $y = x\cos(3x)$ where $0 \le x \le \pi$. Set up but do not evaluate the following integrals:

(a) The length of the curve.

(b) The surface area obtained by revolving about the y-axis.

(c) The surface area obtained by revolving about the line $y = 5$.

5. (6 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$.

6. (10 points) Show that $\sum_{n=2}^{\infty} \left(\frac{cn+1}{n^2-n} \right)$ diverges except for exactly one value c. Compute the sum for that value of c.