Name: ______________________________

Mock Exam #2

Math 231 AD1

1. (21 points) Write **true** if the given statement is always true. Otherwise write **false**.

   (a) ______ If the sequence $\{a_k\}_{k=1}^{\infty}$ converges to a number p with $p > 1$, then the series $\displaystyle\sum_{k=1}^{\infty} a_k$ converges.

   (b) ______ The series $\displaystyle\sum_{n=100}^{\infty} \frac{1}{10n}$ diverges.

   (c) ______ The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{n}}$ diverges.

   (d) ______ The $\displaystyle\lim_{n\to\infty} a_n = 0$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

   (e) ______ If the series $\sum_{n=1}^{\infty} a_n$ diverges, then the sequence $\{a_n\}_{n=1}^{\infty}$ diverges.

   (f) ______ If the series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $\{a_n\}_{n=1}^{\infty}$ converges.

   (g) ______ If the sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing and $a_n \geq \dfrac{1}{n}$ for all positive integers n, then sequence $\{a_n\}_{n=1}^{\infty}$ converges.

2. (8 points) A surface is generated when the curve $y = (x-9)^{1/2}$ with $25 \leq x \leq 45$ is revolved around the y-axis.

   (a) Set up, but do not evaluate, an integral with respect to x which represents the surface area.

   (b) Set up, but do not evaluate, an integral with respect to y which represents the surface area. The limits of integration should be different in parts (a) and (b).

3. (8 points) For which values of x does the following series converge? Your answer should be written in the form $a < x < b$ or $a \leq x \leq b$ for some values of a and b. You do not need to find the sum of the series.

$$1 + \frac{2x}{3} + \frac{4x^2}{9} + \frac{8x^3}{27} + \frac{16x^4}{81} + \frac{32x^5}{243} + \frac{64x^6}{729} + \ldots$$

4. (8 points) The following alternating series converges to a sum S.

$$S = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{8k} = \frac{1}{8} - \frac{1}{16} + \frac{1}{24} - \frac{1}{32} + \dots$$

How many terms do we need to add together to estimate S to within 0.01?

5. (7 points) Determine the sum of the convergent series $\displaystyle\sum_{k=0}^{\infty} \frac{72}{3^{2k+1}}$. Simplify your answer.

6. (7 points) Determine the sum of the convergent series $\displaystyle\sum_{k=0}^{\infty}\left(\frac{k}{k+3}-\frac{k+1}{k+4}\right)$.

7. (7 points) Determine whether the given series converges or diverges. include a carefully written proof to justify your claim. Your proof should sate any convergence or divergence tests you are using, why the test is applicable to this series, and why you can conclude that the series converges or diverges.

$$\sum_{n=1}^{\infty}\frac{n^2-4}{\sqrt{n^6+10}}$$

8. (7 points) Determine whether the given series converges or diverges. include a carefully written proof to justify your claim. Your proof should sate any convergence or divergence tests you are using, why the test is applicable to this series, and why you can conclude that the series converges or diverges.

$$\sum_{k=2}^{\infty} \frac{\ln(k)}{\sqrt{k}}$$

9. (7 points) Determine whether the given series converges or diverges. include a carefully written proof to justify your claim. Your proof should sate any convergence or divergence tests you are using, why the test is applicable to this series, and why you can conclude that the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{\cos(n^2)}{n!}$$