Dubious Explanations	Why there's a problem

$$\boxed{\sum_{n=2}^{\infty} \ln\left(\frac{3n^3-1}{n^3+1}\right)}$$

$$\lim_{t\to\infty}\int_2^t \ln\left(\frac{3x^3-1}{x^3+1}\right)dx \qquad \frac{3n^3-1}{n^3} \leq \frac{3n^3-1}{n^3+1}$$

By p-test $\frac{3n^3-1}{n^3+1}$ converges

1) Haven't justified using the integral test (pos, decr, contin)
2) Inequality is backwards for comparison to $\frac{3n^3-1}{n^3}$
3) p-test doesn't apply here
4) Referring to <u>sequence</u> or <u>series</u> for convergence?

To Fix: Start with Div. Test: $\lim_{n\to\infty} a_n = \ln(3) \neq 0$ So series Diverge.

$$\boxed{\sum_{n=2}^{\infty} \ln\left(\frac{n^2-1}{3n^2+1}\right)} \qquad \ln\left(\frac{n^2-1}{3n^2+1}\right) = \ln(n^2-1) - \ln(3n^2+1)$$

$$= \ln(3)-\ln(13), \ln(8)-\ln(27), \dots \ln(n^2-1)-\ln(3n^2+1)$$

$n^2-1 \neq 13$ or 27 <u>Diverges</u>

1) Does <u>not</u> telescope
2) Just because it doesn't telescope, there's no reason it has to diverge (this one does, but it's not always the case).
3) There's nothing valid to show correct reasoning.

To Fix: Start with Div. Test.

$$\boxed{\sum_{n=0}^{\infty} \frac{1+2^n}{3^n}} \qquad \frac{2^n}{3^n} < \frac{1+2^n}{3^n} \qquad \sum_{n=0}^{\infty}\frac{2^n}{3^n} = \frac{1}{1-\frac{2}{3}} \text{ converges } (r<1)$$

So by comparison test $\sum_{n=0}^{\infty} \frac{1+2^n}{3^n}$ converges

1) Instructions ask you to <u>find the sum for convergent series</u>
2) Comparison test is <u>invalid</u>
   Smaller series converges tells us <u>nothing</u> about larger series $\sum \frac{1+2^n}{3^n}$

To Fix:
- Be careful with comparison tests' initial conditions.
- Split up into 2 series

Limit Issues

$$\lim_{n\to\infty} \frac{1}{3^n} + \left(\frac{2}{3}\right)^n = \text{DNE} \quad \underline{\text{Diverges}}$$

1) It's ok to try Div Test here, but taking a bad limit can <u>really</u> hurt you.
2) The Limit is 0, so Div Test gives no info, & we need to try something else.

To Fix: Be <u>very</u> careful/precise with limits.

$$\ln\left(\frac{3}{13} + \frac{8}{28} + \frac{15}{49} + \dots\right) \longrightarrow \text{goes to } \ln(1) = 0$$

increasing $\longrightarrow$ <u>convergent</u>

1) $\lim_{n\to\infty} a_n = 0$ does <u>NOT</u> imply convergence
2) Check limits carefully, here $\lim_{n\to\infty} a_n = \ln\left(\frac{1}{3}\right) \neq 0$
3) The addition should be on outside of $\ln(\dots)$

Dubious Explanations	Why there's a problem
$\sum_{n=2}^{\infty} \ln\left(\frac{n^2-1}{3n^2+1}\right)$ $\quad \ln\left(\frac{3}{13}\right)+\ln\left(\frac{2}{7}\right)+\ln\left(\frac{15}{33}\right)+\cdots$ $r = \frac{\ln\left(\frac{2}{7}\right)}{\ln\left(\frac{3}{13}\right)}$ $\quad \lvert r\rvert > 1$ Diverges	1) If you state $r =$ "something", then it better be the ratio between <u>all</u> consecutive pairs of terms. 2) This is not a geometric series (nothing raised to the n^{th} power)
$\sum_{n=0}^{\infty} \frac{1+2^n}{3^n}$ $\quad n^{th}$ term test $\lim_{n\to\infty} \frac{1+2^n}{3^n} = \cancelto{0}{\frac{1}{3^n}} + \frac{2^n}{3^n}$ geometric converges to something >0, so divergent	1) limit $= 0$, so we get no info 2) The geometric <u>series</u> converges to something >0, which is not the same as a <u>sequence</u> converging to something >0 (Div Test) To Fix: Correctly state that $\lim_{n\to\infty} \frac{1}{3^n}+\frac{2^n}{3^n}=0$ so Div Test gives no info and move on to Geometric Series Test
$\sum_{n=0}^{\infty} \frac{1+2^n}{3^n}$ $\quad \lim_{n\to\infty} \frac{1+2^n}{3^n} \to \frac{2}{3}$ converges to $\frac{2}{3}$	1) Incorrect limit ($a_n \to 0$) 2) Incorrect conclusion: If $\lim_{n\to\infty} a_n \neq 0$, then <u>Div</u> 3) The limit of the sequence is <u>NOT</u> the sum of the series.