B

Name: Tom

Worksheet #26

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning**.

1. Using the limit of Riemann sums and right endpoints for $\sqrt{x}$ from 0 to 1, compute the following limit by expressing it as an integral: $\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{n^{\frac{3}{2}}}\right)$

$f(x)=\sqrt{x}$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_i = 0 + i\Delta x = \frac{i}{n}$

$f(x_i) = \sqrt{\frac{i}{n}}$

$$\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{n^{3/2}}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\overbrace{\frac{1}{n}}^{\Delta x}\cdot\overbrace{\sqrt{\frac{i}{n}}}^{f(x_i)} = \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \boxed{\frac{2}{3}}$$

this equality comes from identifying the "pieces" on the left

definition of definite integral.

2. Compute the indefinite integral.

(a) $\int x3^{x^2}\,dx = \int x\,3^u \frac{du}{2x} = \frac{1}{2}\int 3^u\,du = \frac{1}{2}\frac{3^u}{\ln 3} + C = \boxed{\frac{3^{x^2}}{2\ln 3} + C}$

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

Recall that $(3^u)' = 3^u \cdot \ln 3$

so $\int 3^u\,du = \frac{3^u}{\ln 3}$

(b) $\int \frac{1-w}{\sqrt{4-w^2}}\,dw = \int \frac{1}{\sqrt{4-w^2}}\,dw + \int \frac{-w}{\sqrt{4-w^2}}\,dw = \boxed{\arcsin\left(\frac{w}{2}\right) + (4-w^2)^{\frac{1}{2}} + C}$

$\int \frac{dw}{2\sqrt{1-\left(\frac{w}{2}\right)^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin(u) + C_1 = \arcsin\left(\frac{w}{2}\right) + C_1$

$u = \frac{w}{2}$

$du = \frac{dw}{2}$

$u = 4-w^2$, $du = -2w\,dw$: $\frac{1}{2}\int \frac{du}{\sqrt{u}} = u^{\frac{1}{2}} + C = (4-w^2)^{\frac{1}{2}} + C$

only need one "+C"

3. Give an integral (you do not need to evaluate it) for the volume of a square pyramid if the base has area 100 and the height of the pyramid is 8.

$$\int_0^8 (\text{side length})^2\,dy = \boxed{\int_0^8 \left(10-\tfrac{5}{4}y\right)^2 dy}$$

when $y=0$, side $=10$
$y=8$, side $=0$ $\Big\}$ side $= 10-\frac{5}{4}y$

4. Compute the maximum value of the function $g(x) = \displaystyle\int_{e^x}^{2e^x} \frac{1}{1+w^2}\,dw$.

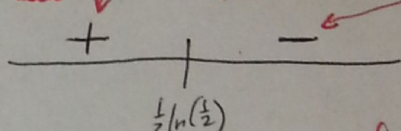
$$g'(x) = \frac{1}{1+(2e^x)^2}\cdot(2e^x)' - \frac{1}{1+(e^x)^2}\cdot(e^x)'$$

$$= \frac{2e^x}{1+4e^{2x}} - \frac{e^x}{1+e^{2x}} \overset{\text{set}}{=} 0 \quad \leftarrow \text{to find max}$$

$$\frac{2e^x}{1+4e^{2x}} = \frac{e^x}{1+e^{2x}} \to 2\cancel{e^x}(1+e^{2x}) = \cancel{e^x}(1+4e^{2x}) \to 2+2e^{2x} = 1+4e^{2x}$$

$$\to 1 = 2e^{2x} \to 2x = \ln\left(\tfrac{1}{2}\right) \to x = \tfrac{1}{2}\ln\left(\tfrac{1}{2}\right)$$

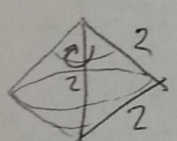
for $x=-1$ ~~[illegible]~~ (try $x=0$) $\frac{2}{5}-\frac{1}{2}<0$



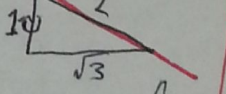
verify that it is a max

Max occurs at $\frac{1}{2}\ln(\frac{1}{2})$, but the actual max value is $g\left(\frac{1}{2}\ln(\frac{1}{2})\right) = \arctan(\sqrt{2}) - \arctan\left(\frac{1}{\sqrt{2}}\right)$

5. Given an integral (you do not need to evaluate it) for the volume obtained by rotating an equilateral triangle with side length 2 about one of its sides.



Volume $= 2\cdot$(volume of top half)



$x = \sqrt{3}-\sqrt{3}y$
or
$y = 1-\frac{1}{\sqrt{3}}x$

(washers) $\boxed{\displaystyle\int_0^1 \pi\underbrace{\left(\sqrt{3}-\sqrt{3}y\right)}_{r}^2 dy}$

(shells) $\boxed{\displaystyle\int_0^{\sqrt{3}} 2\pi \underset{r}{x}\underbrace{\left(1-\tfrac{1}{\sqrt{3}}x\right)}_{h} dx}$