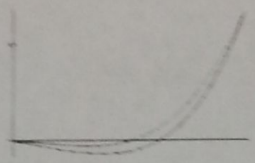
Name: _____

Worksheet #25

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning**.

1. The shaded region is bounded by the curves $y = x^4 - 2x$ and $y = 3x^3 - 3x^2$ over $[0, 2]$



These functions can't be written as $x = \ldots$, so we need to use "dx" integrals in each part.

(a) Give an integral (do not evaluate) for the area of the region.

(b) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $y = -3$.

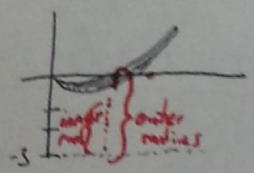
(c) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $x = 8$.

a) To determine top/bottom, plug in $x=1$: $3(1)^3 - 3(1)^2 > (1)^4 - 2(1)$, so $3x^3 - 3x^2$ is top & $x^4 - 2x$ is bottom

$$\int_0^2 (3x^3 - 3x^2) - (x^4 - 2x)\,dx$$

b) $\int_a^b \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\,dx = \int_0^2 \pi(3x^3 - 3x^2 + 3)^2 - \pi(x^4 - 2x + 3)^2\,dx$

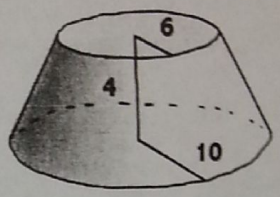
washers



c) $\int_0^2 (2\pi \cdot r \cdot h)\,dx = \int_0^2 2\pi \underbrace{(8 - x)}_{\text{radius}} \underbrace{(3x^3 - 3x^2 - x^4 + 2x)}_{\text{height (from (a))}}\,dx$

shells

2. (*8 points*) A tank is shaped like a piece of a right circular cone whose base radius $b = 10$ and top radius $a = 6$ and overall height is 4. Write down the integral (you do not need to evaluate it) for the work required to pump the water out the top of the tank. (Use 9.8 m/s^2 for g. Use 1000 kg/m^3 as the weight density of water.)



Work = force · distance

= (area · thickness) · density · acceleration · distance

(area · thickness) = volume

Using y to be the distance water must be pumped to the top,

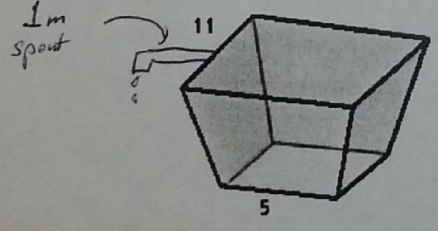
when $y=0$, $r=6$ & when $y=4$, $r=10$.

So the radius, $r(y) = 6+y$

$$W = \int_0^4 \pi(6+y)^2 \cdot dy \cdot 1000 \cdot 9.8 = \int_0^4 9800\pi y(6+y)^2\,dy$$

$\pi(6+y)^2 \cdot dy$ = volume; 1000 = dens; 9.8 = accel.; y = distance

3. A tank shaped like an inverted frustum of a pyramid with square base of side length 5 m. along the base and 11 m. along the top with height 2 m. is full of water. Write down the integral (you do not need to evaluate it) for the work required to pump the water out of a spout 1 m. long. (Use 9.8 m/s^2 for g. Use 1000 kg/m^3 as the weight density of water.)



Again, we use $y=0$ to be the top and $y=2$ to be the bottom

side length: $s(y) = 11-3y$

$$\int_0^2 (\text{Area} \cdot \text{dens} \cdot \text{gravity} \cdot \text{dist})\,dy = \int_0^2 (11-3y)^2 1000(9.8)(y+1)\,dy$$

(Area · dens · gravity · dist)dy = volume