Name: Tom

F

Worksheet #23

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning**.

1. Calculate the following definite integrals.

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(x)\cos(x)}{\sqrt{1-\sin x}}\,dx$ (b) $\displaystyle\int_0^{\frac{\pi}{4}} \tan x\,dx$ (c) $\displaystyle\int_0^4 x^3\sqrt{x^2+2}\,dx$

Change limits!

$u = 1-\sin(x)$

$du = -\cos(x)\,dx$

$\boxed{\begin{array}{l} x=0 \to u=1 \\ x=\frac{\pi}{2} \to u=0 \end{array}}$

a) $\displaystyle\int_1^0 \frac{\sin(x)\cos(x)}{\sqrt{u}}\frac{du}{-\cos(x)} = -\int_1^0 \frac{\sin(x)}{\sqrt{u}}\,du = -\int_1^0 \frac{1-u}{\sqrt{u}}\,du = \int_0^1 u^{-\frac{1}{2}} - \sqrt{u}\,du = 2\sqrt{u} - \frac{2}{3}u^{3/2}\Big|_0^1$

$\displaystyle = \boxed{\left(2-\frac{2}{3}\right) - (0-0)}$

$\sin(x) = 1-u$ rewrite leftover $\sin(x)$

flip limits to get rid of minus sign

b) $\displaystyle\int_0^{\frac{\pi}{4}} \tan(x)\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin(x)}{\cos(x)}\,dx = \int_1^{\frac{\sqrt{2}}{2}} \frac{\sin(x)}{u}\frac{du}{-\sin(x)} = -\int_1^{\frac{\sqrt{2}}{2}} \frac{du}{u} = \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{u}\,du = \ln|u|\Big|_{\frac{\sqrt{2}}{2}}^1 = \boxed{-\ln\left(\frac{\sqrt{2}}{2}\right)}$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$\boxed{\begin{array}{l} x=0 \to u=1 \\ x=\frac{\pi}{4} \to u=\frac{\sqrt{2}}{2} \end{array}}$ Change limits

c) $\displaystyle\int_0^4 x^3\sqrt{x^2+2}\,dx = \int_2^{18} x^3\sqrt{u}\,\frac{du}{2x} = \int_2^{18} \frac{1}{2}x^2\sqrt{u}\,du = \int_2^{18} \frac{1}{2}(u-2)\sqrt{u}\,du = \frac{1}{2}\int_2^{18} u^{3/2} - 2\sqrt{u}\,du$

rewrite leftovers $(x^2 = u-2)$

$\displaystyle = \frac{2}{10}u^{5/2} - \frac{4}{3}u^{3/2}\Big|_2^{18}$

$u = x^2+2$

$du = 2x\,dx$

$\boxed{\begin{array}{l} x=0 \to u=2 \\ x=4 \to u=18 \end{array}}$ New limits

$\displaystyle = \boxed{\frac{2}{10}(18)^{5/2} - \frac{4}{3}(18)^{3/2} - \left(\frac{2}{10}(2)^{5/2} - \frac{4}{3}(2)^{3/2}\right)}$

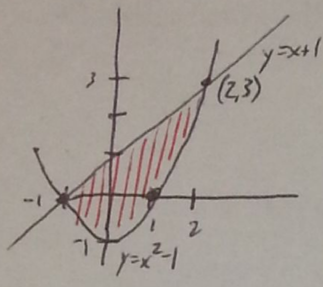
These problems only ask for the setup of the integral, but in case you want to practice integration, I have included the full solution.

2. Write an integral for the area bounded by $y = x + 1$ and $y = x^2 - 1$.

Find intersection points: Set $x+1 = x^2-1 \rightarrow x^2-x-2=0 \rightarrow (x-2)(x+1)=0 \rightarrow x=-1,2$ Limits

$$\int_{-1}^{2} \underbrace{(x+1)}_{\text{top curve}} - \underbrace{(x^2-1)}_{\text{bottom curve}}\,dx = \int_{-1}^{2} -x^2+x+2\,dx = -\frac{1}{3}x^3+\frac{1}{2}x^2+2x\Big|_{-1}^{2}$$

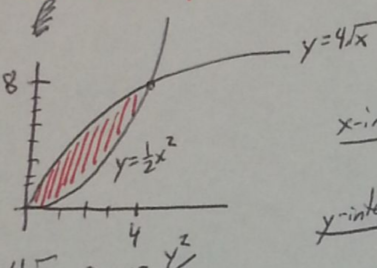
$$= \boxed{-\frac{1}{3}(8)+\frac{1}{2}(4)+4-\left(\frac{1}{3}+\frac{1}{2}-2\right)}$$



3. Consider the region bounded by $y = 4\sqrt{x}$ and $y = \frac{x^2}{2}$. Write two integrals, one with respect to x and the other to y, that express the area of this region.

Intersection points: $4\sqrt{x} = \frac{x^2}{2} \rightarrow x^2 - 8\sqrt{x} = 0 \rightarrow \sqrt{x}(x^{3/2}-8)=0 \rightarrow x=0,4$ (x-limits)

Plugging into our functions gives us y-limits: $y=0,8$



x-integral: $\int_0^4 \underbrace{4\sqrt{x}}_{\text{top}} - \underbrace{\frac{x^2}{2}}_{\text{bottom}}\,dx = \frac{8}{3}x^{3/2} - \frac{x^3}{6}\Big|_0^4 = \boxed{\frac{8}{3}(4^{3/2}) - \frac{4^3}{6}}$

y-integral: $\int_0^8 \underbrace{\sqrt{2y} - \frac{y^2}{16}}_{\text{right} - \text{left}}\,dy = \frac{2\sqrt{2}}{3}y^{3/2} - \frac{y^3}{48}\Big|_0^8 = \boxed{\frac{2\sqrt{2}}{3}(8)^{3/2} - \frac{8^3}{48}}$

simplify to the same #

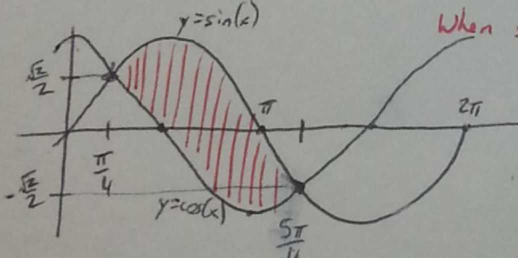
$\sqrt{2}\cdot\sqrt{y}$

$y = 4\sqrt{x} \rightarrow x = \frac{y^2}{16}$

$y = \frac{x^2}{2} \rightarrow x = \sqrt{2y}$

4. Write an integral for the area bounded by $y = \sin x$ and $y = \cos x$ from $\frac{\pi}{4}$ to $\frac{5\pi}{4}$.

When $x = \frac{\pi}{4}, \frac{5\pi}{4}$, $\sin(x) = \cos(x)$, & there are no intersection points between these values (by looking at the graph)



$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \underbrace{\sin(x)}_{\text{top}} - \underbrace{\cos(x)}_{\text{bottom}}\,dx = (-\cos(x) - \sin(x))\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= -\cos\left(\tfrac{5\pi}{4}\right) - \sin\left(\tfrac{5\pi}{4}\right) + \cos\left(\tfrac{\pi}{4}\right) + \sin\left(\tfrac{\pi}{4}\right)$$

$\boxed{\text{DO simplify "nice" trig values}}$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2}}$$