Pythagoras

Name: ____

Worksheet #22

Math 221

Instructions. Put the your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to **explain your reasoning**.

you may notice that we know $F(x) = \arctan(x)$, but FTC part I does not require knowing $F(x)$

1. Compute $\dfrac{d}{ds}\displaystyle\int_{\cos s}^{s^2+2} \frac{1}{x^2+1}\,dx. = \frac{d}{ds}\left(F(s^2+2) + F(\cos s)\right) = f(s^2+2)\cdot 2s - f(\cos(s))\cdot(-\sin s)$

$$= \boxed{\frac{1}{(s^2+2)^2+1}\cdot 2s - \frac{-\sin(s)}{\cos^2(s)+1}}$$

Fundamental Theorem of Calc Part I

2. A point is moving along the x-axis with acceleration $45t + 1$ at time t. If its initial velocity is 27 how far does the particle move from time $t = 1$ to time $t = 3$?

$a(t) = 45t + 1$

$v(t) = \int a(t)\,dt = \frac{45}{2}t^2 + t + C$

$v(0) = 27 = \frac{45}{2}(0^2) + 0 + C \Rightarrow C = 27$

initial velocity $= 27$

$$\int_1^3 v(t)\,dt = \int_1^3 \frac{45}{2}t^2 + t + 27\,dt$$

$$= \frac{45}{2\cdot 3}t^3 + \frac{1}{2}t^2 + 27t\Big|_1^3$$

integrate velocity over the time interval to get change in dist.

$$= \boxed{\frac{15}{2}(3^3) + \frac{3^2}{2} + 27(3) - \left(\frac{15}{2}\cdot 1 + \frac{1}{2} + 27\right)}$$

yuck! I certainly wouldn't want to bother ~~simplifying~~ simplifying this.

3. A patient is experiencing blood loss at a rate of $\frac{2}{1+t^2}$ pints per minute at time t (as t increases, the blood loss slows due to decreased pressure). Approximately how long before the patient loses 3 pints of blood after $t = 0$?

rate $= \frac{2}{1+t^2}$

total loss $= \int \frac{2}{1+t^2}\,dt$

the unknown is upper time limit

want

$$\int_0^x \frac{2}{1+t^2}\,dt = 2\arctan(x) - 2\arctan(0) = 3$$

$$\Rightarrow \arctan(x) = \frac{3}{2}$$

$$\boxed{x = \tan\left(\frac{3}{2}\right)}$$

4. Compute the indefinite integrals using the indicated substitution.

(a) $\int 2x\sqrt{1+x^2}\,dx \quad u = 1+x^2$

(b) $\int \frac{2+\ln(x)}{x}\,dx \quad u = \ln x$

(c) $\int \frac{2x}{1+x^4}\,dx \quad u = x^2$

(d) $\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\,dx \quad u = \sqrt{x}$

a) $u = 1+x^2$

$du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

$\int 2x\sqrt{1+x^2}\,dx = \int \cancel{2x}\sqrt{u}\,\frac{du}{\cancel{2x}} = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{3}(1+x^2)^{3/2} + C}$

b) $u = \ln x$

Note $u = 2 + \ln x$ would also work. There would be a different "+C"

$du = \frac{1}{x}\,dx \Rightarrow dx = x\,du$

important NOT to write $\ln x^2$ or $\ln(x^2)$

$\int \frac{2+\ln(x)}{x}\,dx = \int \frac{2+u}{\cancel{x}}\cdot \cancel{x}\,du = \int 2+u\,du = 2u + \frac{1}{2}u^2 + C = \boxed{2\ln(x) + \frac{1}{2}(\ln x)^2 + C}$

c) $u = x^2$

$du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

$\int \frac{\cancel{2x}}{1+u^2}\,\frac{du}{\cancel{2x}} = \int \frac{1}{1+u^2}\,du = \arctan(u) + C = \boxed{\arctan(x^2) + C}$

d) $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow dx = 2\sqrt{x}\,du = 2u\,du$

$\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\,dx = \int \frac{\sin(u)}{\cancel{2u}}\cdot \cancel{2u}\,du = \int \sin(u)\,du = -\cos(u) + C = \boxed{-\cos(\sqrt{x}) + C}$